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CONTENTS

NUMBER I

NUMBER II

vi
CONTENTS

ON THE AXIAL ROTATION OF STARS

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ABSTRACT

Broad and *shallow* absorption *lines* in stellar spectra are shown to be *due to axial rotation*. The evidence is based upon the following facts: (*a*) the *broadening depends upon wave-length*, as required by the Doppler principle; (*b*) in *spectroscopic binaries* there is a distinct *correlation* between *line width* on one side and *period and amplitude* on the other; and (*c*) as was shown by Elvey, the contours of "*dish-shaped*" lines agree well with the theoretical shapes of lines of rapidly rotating stars.

A survey of the spectra of stars of various types shows that *rapid rotation is peculiar to the earliest spectral classes.* The B- and A-stars display the greatest tendency toward rapid rotation. In the F's the proportion of rapidly rotating stars is smaller, and in classes G, K, and M no cases of rapid rotation have been observed in single stars.

The effect of *rotation* upon the contours of *components of spectroscopic binaries* is discussed. In *α* Virginis the stronger component is found to rotate with an equatorial velocity of 200 km/sec., while for the fainter component the rotational velocity is not more than about 50 km/sec. This indicates a marked *difference in the size of the components.* The ratio of the radii is roughly 4 to 1.

The contours of lines in *η* Ursae Majoris suggest an equatorial velocity of about 200 km/sec. This *single star* is thus *similar to* the stronger component of *α Virginis.* Both stars are found to be *stable*, but the characteristic quantity $\omega^2/2\pi\gamma\rho$ is not far from the critical value where the Maclaurin ellipsoids of rotation lose stability. It is suggested that there is a *transition between close spectroscopic binaries and rapidly rotating single stars* in the earliest spectral classes. Observations do not establish the direction in which this transition proceeds.

I. INTRODUCTION

Many theoretical discussions of the origin of spectroscopic binaries and of other cosmogonic problems have been based upon the dynamical consequences of rapid axial rotation of single stars. The fission theory originated by Sir George Darwin and developed by Sir James Jeans[1] is built upon the assumption that stars lose energy

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[1] *Astronomy and Cosmogony*, Cambridge, 1928.

by radiation and are therefore forced to contract. In doing so they must preserve their angular momentum and this can be accomplished only if the angular velocity is increased. A simple computation shows that an average star must rotate at a very high speed before instability can become dangerous. Not until the equatorial velocity of a typical B-type star has reached a value of the order of several hundred kilometers per second is there any danger of break-up. Indeed, F. R. Moulton[1] has shown that the sun must contract until its equatorial radius is 11 miles and its mean density 3×10^{13} on the water standard, before the Maclaurin ellipsoids of rotation become unstable. Moulton concludes that "the oblateness of the sun can never approach that for which the Jacobian figures of equilibrium branch." The difficulty obviously lies in the fact that for the sun with its slow rate of rotation the angular momentum is small, and that an enormous contraction would be required to increase the angular velocity to the critical value.

The problem would assume an entirely different aspect if it could be shown observationally that there are in existence stars, with normal average densities, for which the rotational velocities are large, so large indeed that the characteristic value of $\omega^2/2\pi\gamma\rho$ is not far removed from the critical point where instability sets in.

Until recently such observational evidence was completely lacking. It seems to have been the opinion of many of the leading observational astronomers that stars rotate with equatorial velocities of a few kilometers per second at the most. The relative velocity at the equatorial limbs of the sun amounts to only 3.9 km/sec., and by analogy it was inferred that similar conditions must hold for the stars in general.[2]

[1] *Carnegie Institution of Washington, Publication* No. 107, 150, 1909. Also *Astrophysical Journal,* **29,** 1, 1909.

[2] J. Evershed says in *Monthly Notices of the Royal Astronomical Society,* **82,** 395, 1922: ". . . . The angular speed [of Sirius] will be over three times that of the Sun, a complete rotation taking eight days or less. This high speed of rotation seems improbable considering that Sirius is in an earlier stage of evolution, and is therefore less condensed than the Sun. . . . ." Similarly J. A. Carroll expresses the opinion (*ibid.,* **88,** 555, 1928): ". . . . We are thus able to say that no stars have been found rotating with an equatorial velocity much greater than say 50 km/sec. and that . . . . the rotational speed must have been more nearly of the order of 10 km/sec. or less." The same idea is stated by H. C. Vogel (*Astronomische Nachrichten,* **90,** 75, 1877): "Ein Aequator-

However, in the light of observations made within the past few years[1] our early conceptions of this subject must be radically changed. It appears that rapidly rotating stars are by no means rare exceptions. They are quite common in the earlier spectral classes, and their equatorial velocities occasionally exceed 200 or even 250 km/sec. Such velocities are characteristic for many single stars, as well as for spectroscopic binaries of short period and large amplitude. This naturally leads to the inference that there is a continuous transition between single stars having rapid axial rotations and close spectroscopic binaries of short period.

The observations do not indicate in which direction this transition proceeds. It remains uncertain whether a rapidly rotating single star breaks up into a binary (as would follow from the fission theory), or whether the components of a binary, by falling together, give rise to a rapidly rotating single star. Nevertheless, it is of considerable cosmogonic interest that the stars of earliest spectral classes—A, B, and probably O—reveal the greatest tendency toward rapid rotation. It is in these types, then, that we should expect the binaries to originate, if we were to accept the fission theory. The rotational velocities of many single stars are of the same order of magnitude as those shown by components of the closest-known spectroscopic binaries, and are not much inferior to those predicted theoretically for the "break-up" of a single star.

This paper is not directly concerned with the fission theory. It is well known that Professors F. R. Moulton[2] and W. D. MacMillan[3] have raised a number of important objections to it, and it is not here possible to pursue the problem of the origin of double stars farther. Neither do we touch upon the question as to how the rapid rotations in single stars have originated. It is not easy to reconcile

punkt von a Aquilae . . . . würde die immer noch ansehnliche Geschwindigkeit von 25 geogr. Meilen haben. Es erscheinen diese Geschwindigkeiten, zumal im Vergleich mit der Rotationsgeschwindigkeit unserer Sonne (Aequatorpunkt o.27 geogr. Meilen) als im hohen Grade unwahrscheinlich." See also the article by K. Walter in *Die Sterne*, **10**, 9, 1930.

[1] Shajn and Struve, *Monthly Notices of the Royal Astronomical Society*, **89**, 222, 1929. This paper contains references to earlier investigations on the subject of stellar rotation; C. T. Elvey, *Astrophysical Journal*, **71**, 221, 1930; *ibid.*, **70**, 152, 1929.

[2] *Op. cit.*, pp. 156, 160. [3] *Science*, **62**, 69, 1925.

the facts with the contraction hypothesis, and we must at present be content with the purely observational result that rapid rotations can and do occur.

II. THE INTERPRETATION OF LINE CONTOURS

The evidence concerning rotation depends primarily upon the interpretation of the contours of stellar absorption lines. It is now known that there are three major factors[1] which influence the shape of these contours, viz., abundance of atoms, molecular Stark effect, and axial rotation.

a) The contour of an absorption line in a scattering atmosphere without collisions is known from theoretical considerations. Observations have shown that this "Unsöld contour" is closely obeyed by the majority of spectral lines in late-type stars. Small corrections due to the influence of collisions have been computed by Unsöld and tend to make the agreement between observation and theory even better. The abundance of atoms in the right state to absorb a given line depends mainly upon the temperature and pressure of the reversing layer. The actual percentage of occurrence of any one element in a stellar atmosphere is remarkably constant along the spectral series. For many lines the effect of absolute magnitude is very small and can be neglected, in which case the total amount of energy absorbed in any one line is a function of spectral type alone.

b) The hydrogen lines show anomalous contours in all spectral classes. The same is true of the helium lines in many B-type stars. These anomalies are due to Stark effect, produced by the electrical fields of neighboring ions. The resulting broadening of the lines is not the same in all stars; it varies with the pressure and is consequently related to absolute magnitude. However, as in the case of

[1] There are many physical factors which influence the contour of a line in the laboratory. Of these only the molecular Stark effect is believed to be effective in stellar atmospheres. Doppler effect due to temperature agitation is probably not appreciable (Vasnecov, *Vestnik Kral. Ces. Spol. Nauk*, **2**, 1927). Most of the other effects observed in the laboratory are due to direct interaction of the atoms. Such effects are improbable in the stars on account of the low pressures in the reversing layers. The Stark effect alone is rendered appreciable because of the high state of ionization. See also p. 5, n. 2.

abundance, the effect of absolute magnitude is comparatively small, and can be neglected if we consider only the major features of a stellar spectrum.

c) A considerable number of stars belonging to classes O, B, and A, and occasionally to class F, show broad and hazy lines for all elements, wholly different in shape from the narrow and deep contours produced by the normal absorption coefficient of Unsöld. C. T. Elvey[1] has shown that these "dish-shaped" lines agree with the contours computed by G. Shajn and the writer for rapidly rotating stars.

In section VI we shall compare the contours of certain lines of a single star (η Ursae Majoris) with those of a spectroscopic binary (α Virginis). The agreement is so satisfactory that there remains no doubt that both are produced by the same cause. Since in the binary this cause is known to be rotation[2] the single star, too, must be rapidly rotating.

III. ROTATION AND SPECTRUM

It has been customary at the Yerkes Observatory to classify all spectra according to the character of the lines. This classification was originally introduced by Professor E. B. Frost in order to provide a rough criterion as to the precision to be expected from measurements of radial velocity. Since "poor" lines are invariably dish-shaped, and since even the absence of strong lines can be interpreted as being due to excessive diffuseness, it appears that this classification is a good measure of rotation, or rather of the component of the rotational velocity in the line of sight. The distribution of all stars of types B and A observed at Yerkes[3] is shown in Tables I and II.

[1] *Astrophysical Journal*, **70**, 152, 1929.

[2] This follows from the correlation between line width and a function of period and amplitude (*Monthly Notices of the Royal Astronomical Society*, **89**, 225, 1929):

$$v = \text{const.} \frac{r \cdot K}{P^{\frac{1}{3}}} .$$

Evershed has suggested that wide and hazy lines may be due to Doppler effect caused by violent convection currents (*ibid.*, **82**, 395, 1922). This may be a contributory cause, but it does not explain the correlation just noted. The latter strongly suggests that we are dealing principally with Doppler effect due to rotation.

[3] *Astrophysical Journal*, **64**, 9, 1926; *Publications of the Yerkes Observatory*, **7**, Part I, 10, 1929.

It will be seen that the proportion of stars classified as "few, poor" is very great among the B's as well as among the A's. The number of F stars observed is too small to justify a complete tabulation, but from the existing spectrograms it appears that the proportion of dish-shaped lines is much smaller than in the B's and A's. Finally, in spectral classes G, K, and M we have never observed any stars having very wide lines. This refers mostly to the giants, since there have been only few dwarfs of late types on our program.

TABLE I

B STARS

Character of Lines	Number of Stars
Many, good	4
Few, good	59
Many, fair	17
Few, fair	101
Many, poor	7
Few, poor	139

TABLE II

A STARS

Many, good	118
Few, good	37
Many, fair	71
Few, fair	59
Many, poor	35
Few, poor	180

It seems safe to say that rotational speed is a function of spectral type, the fastest rotations occurring in the earliest types.

In the following sections we shall assume that all dish-shaped lines are caused by rotation alone and that, furthermore, broadening due to abundance and Stark effect is constant within any given spectral subdivision. That this assumption is permissible results from the fact that among the stars of early types variations due to dish-shaped character are by far the most important of any observed. Minor differences in abundance or pressure broadening depending upon the absolute magnitude can be neglected if we limit our discussion to stars having very flat and broad lines.[1]

[1] The assumption that Stark effect does not vary much within any one spectral subdivision is a modification of certain ideas put forward in my first article on Stark

IV. LINE WIDTH AND WAVE-LENGTH

Consider a rapidly rotating star. The lines are widened by amounts depending upon the projection of the equatorial velocity upon the line of sight. According to Doppler's principle the amount of widening is

$$\Delta\lambda = \lambda \frac{v \cdot \sin i}{c} . \tag{1}$$

But the wave-length in A.U. is related to the readings on the plate in millimeters by Hartmann's formula

$$\lambda - \lambda_0 = \frac{k}{S_0 - S} . \tag{2}$$

Differentiating this we get

$$\Delta\lambda = \frac{k}{(S_0 - S)^2} \cdot \Delta S = \frac{(\lambda - \lambda_0)^2}{k} \Delta S .$$

Substituting this into (1) we find

$$\Delta S = \frac{\lambda \cdot v \cdot \sin i \cdot k}{c(\lambda - \lambda_0)^2} . \tag{3}$$

Consequently the width of a line broadened by Doppler effect should show the following proportionality:

$$\Delta S \infty \frac{\lambda}{(\lambda - \lambda_0)^2} . \tag{4}$$

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effect (*Astrophysical Journal*, **69**, 185, 1929). It was suggested there that the diffuse appearance of the hydrogen and helium lines in many stars might be due to electric fields. It now appears that this explanation is correct for the hydrogen lines only (C. T. Elvey, *ibid.*, **71**, 191, 1930; E. T. R. Williams, *Harvard College Observatory Circular*, No. 348, 1930). The variation in width of the helium lines due to Stark effect is comparatively small, though real (cf. the contours in γ Pegasi and in 67 Ophiuchi), and the dish-shaped character must be ascribed almost wholly to rotation. The existence of an effect of absolute magnitude for the hydrogen lines is largely due to changes in Stark effect, since here Stark effect is very pronounced and rotation is little effective. In the helium lines the relationship of character (*s* or *n*) to absolute magnitude must be due to a real tendency of the more luminous stars to rotate slower than the less luminous stars.

OTTO STRUVE

For single-prism spectrograms taken with the Bruce spectrograph of the Yerkes Observatory the constant λ_0 is approximately equal to 2300 A.

I have measured the line widths in a microphotometer tracing of a spectrogram of α Virginis. This is a binary showing two very unequal components, but the particular plate selected for measurement shows only one set of lines, the fainter component being superposed over the stronger.

The results of the measurements, given in column 3, clearly show a relationship to λ in rough agreement with formula (4). However,

TABLE III

RELATION BETWEEN LINE WIDTH AND WAVE-LENGTH

Wave-Length	Central Intensity	Width	Corrected Width
3926	0.11	7.0 mm	7.0 mm
4009	.06	5.5	6.2
4026	.13	7.0	5.3
4121	.09	5.0	5.0
4144	.10	5.0	5.0
4388	.09	5.0	5.0
4471	.13	7.0	5.0
4481	.04	3.5	4.6
4552	0.05	3.5	4.2

better results can be obtained if small corrections are introduced to take care of differences in central intensity. By a simple graphical procedure all line widths were reduced to a uniform central intensity of 0.10. The values thus obtained are given in the last column. The decrease in width with wave-length is very nearly that required by formula (4). The evidence is in favor of the rotation hypothesis, since it affects all lines.

V. APPLICATION TO THE STUDY OF SPECTROSCOPIC BINARIES

Figure 1 shows a microphotometer tracing of the spectrum of 67 α Virginis near maximum separation of the lines. Two components are visible for each of the two lines, λ 4472 and λ 4481. It is at once obvious that the shapes of the lines are not the same. The violet components of both lines are shallow and very broad, while

the red components are much narrower.[1] Figure 3 shows the contours evaluated by means of the sensitometer intensities given at the right margin of Figure 1.
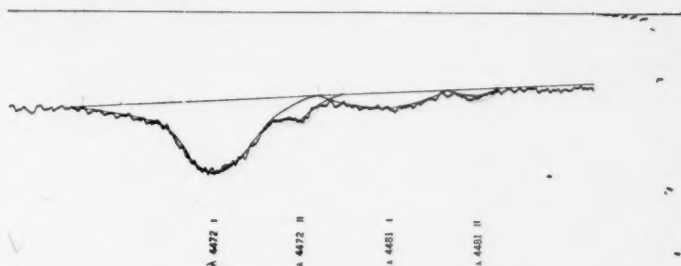
FIG. 1.—Microphotometer tracing of Plate R 1788 of α Virginis, taken on April 24, 1930, at 5^h 35^m U.T. The sensitometer intensities, of which only nine are shown near the right-hand margin, correspond to the following differences in intensity, expressed in stellar magnitudes (from the top): 0.00; 0.39; 0.70; 1.13; 1.47; 1.93; 2.26; 2.75; 3.11; 3.45; 3.86; 4.23; 4.48; 5.06.
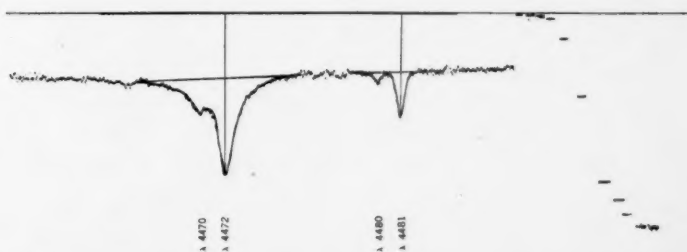
FIG. 2.—Microphotometer tracing of Plate R 1599 of γ Pegasi. Note the sharpness of the lines. The forbidden helium line at λ 4470 is visible to the right of the strong *(left)* helium line λ 4472. The line at λ 4480 is due to *Al* III. The plate was taken on September 21, 1929, at 4^h38^m U.T.

The conclusion is obvious: One component of the binary is in rapid rotation while the other is not. This is a rather striking result and it leads to several interesting consequences.

The spectroscopic elements of α Virginis, as derived by R. H. Baker,[2] are:

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[1] This is true of all other lines not shown in the figure. The narrow component is particularly well visible in *He* 4713. It has also been measured in *He* 4026 and *He* 4388.

[2] *Publications of the Allegheny Observatory*, **1**, 65, 1909. Measures of our spectrograms agree well with Baker's orbit.

Velocity of system................. $+1.6$ km/sec.
Period........................ 4.01416 days
Eccentricity...................... 0.10
Time of periastron passage......... 1908 Jan. 14. 846
Longitude of periastron............ 328°
K_1............................... 126.1 km/sec.
K_2............................... 207.8 km/sec.
$a_1 \sin i$........................... 6,930,000 km
$a_2 \sin i$........................... 11,400,000 km
$m_1 \sin^3 i$........................... 9.6⊙
$m_2 \sin^3 i$........................... 5.8⊙

$\lambda 4472$ $\lambda 4481$
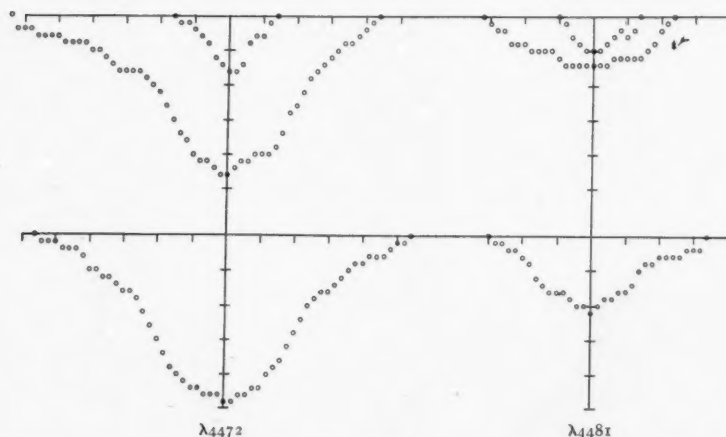
FIG. 3.—Observed contours of lines in α Virginis (top) and η Ursae Majoris (bottom). The contours of both components of α Virginis are shown. One unit in the abscissa corresponds to 1.28 A.U., and one unit in the ordinate to an absorption of 0.05 of the continuous spectrum.

The orbital period is so short that the rotational periods are doubtless equal to it. Consequently the two components have identical angular velocities. Since

$$v_1 = r_1 \omega_1 \sin i ,$$
$$v_2 = r_2 \omega_2 \sin i ,$$
$$\omega_1 = \omega_2 ,$$

the disparity in the v's must mean a disparity in the radii. We arrive at the conclusion that the stronger component of α Virginis (violet in Fig. 1) has the greater diameter. Information concerning stellar radii can thus be obtained by a purely spectroscopic method.

The binary is not resolved visually; consequently the light of both components enters the slit of the spectrograph simultaneously. Suppose the radial velocities are such that the two components are completely resolved. The continuous spectrum of one star will overlap the line of the other. Let the intensities of the continuous spectra outside the lines be i_1 and i_2 and let the real intensities within the two lines, in the absence of overlapping, be given by

$$j_1 = f_1(\lambda) \quad \text{and} \quad j_2 = f_2(\lambda) .$$

The spectral types of the two components are the same. Consequently, if there were no rotation, we should have

$$\frac{j_1}{i_1} = \frac{j_2}{i_2} .$$

It should be noted here that the contours are usually expressed in units of the intensity of the continuous spectrum. Hence the fractions in the foregoing expression.

The rotational effect may not be the same in both stars, and the foregoing equality will not, in general, be fulfilled. But rotation changes only the shape of the contour, leaving the total amount of absorbed energy unaffected. Consequently the integrals taken over the functions j_1/i_1 and j_2/i_2 should be identical:

$$A = \int_{-\infty}^{+\infty} \left(1 - \frac{j_1}{i_1} \right) d\lambda = \int_{-\infty}^{+\infty} \left(1 - \frac{j_2}{i_2} \right) d\lambda .$$

In reality we do not observe j_1 and j_2 separately. Both components are photographed simultaneously, and the plate records the combined effect of the lines and of the continuous spectra. As a result of this overlapping we observe for the two components the quantities A_1 and A_2, which are not, in general, identical:

$$A_1 = \int_{-\infty}^{+\infty} \left(1 - \frac{j_1 + i_2}{i_1 + i_2} \right) d\lambda = \frac{i_1}{i_1 + i_2} A , \tag{5}$$

$$A_2 = \frac{i_2}{i_1 + i_2} A . \tag{6}$$

From these

$$\frac{A_1}{A_2} = \frac{i_1}{i_2} .$$

The observed areas of the contours of the two components are proportional to the intensities of their continuous spectra. Since the spectral types are the same, we obtain the difference in absolute magnitude:

$$M_1 - M_2 = -2.5 \log \frac{i_1}{i_2} = -2.5 \log \frac{A_1}{A_2} . \tag{7}$$

By means of this formula we compute the values of ΔM if A_1 and A_2 are known from the observations.

We shall now reconstruct from the observed contours the real contours j_1 and j_2, which the lines would have had if overlapping with the continuous spectrum did not occur. It follows from (5) and (6) that this is accomplished by multiplying the ordinates of the observed contours by $(1+[A_2/A_1])$ and by $(1+[A_1/A_2])$, respectively.

The numerical evaluation of A_1 and A_2 from the curves of Figure 3 gives roughly:

$$\frac{A_1}{A_2} = 8.8 .$$

Consequently[1]

$$M_2 - M_1 = 2.4 \text{ mag.}$$

Figure 4 shows the corrected contours as they would have been without overlapping. The difference in shape is very marked, confirming that the rotational effect is not identical in the two components.

We now proceed to evaluate the rotational velocity. Let us assume that the shape of the line λ 4472, not affected by rotation, is that obtained by J. Pauwen[2] for the star γ Pegasi (Fig. 2). The

[1] It may be noted that the lack of broadening of the fainter component, its greater "compactness", makes it possible to observe it even though $(M_1 - M_2)$ is rather large. If the second component were as broad and diffuse as the primary it could not have been seen on the background of the continuous spectrum. The actual value obtained, 2.4 mag. should be considered as a rough approximation only. It is probable that since the line is very faint even on our best plates, the measured area A_2 is slightly too small. That this is so may be seen from the fact that a line can be lost completely if it is too diffuse and broad.

[2] *Astrophysical Journal*, **70**, 263, 1929.

spectral types of α Virginis and γ Pegasi are approximately the same. Measurement of the areas shows that the line in γ Pegasi is not as strong as the line in α Virginis. In order to make the two areas identical, we multiply all ordinates of the contour for γ Pegasi by 1.8. The resulting contour is shown in Figure 5. We proceed in a manner similar to that used by Shajn and the writer, which was also
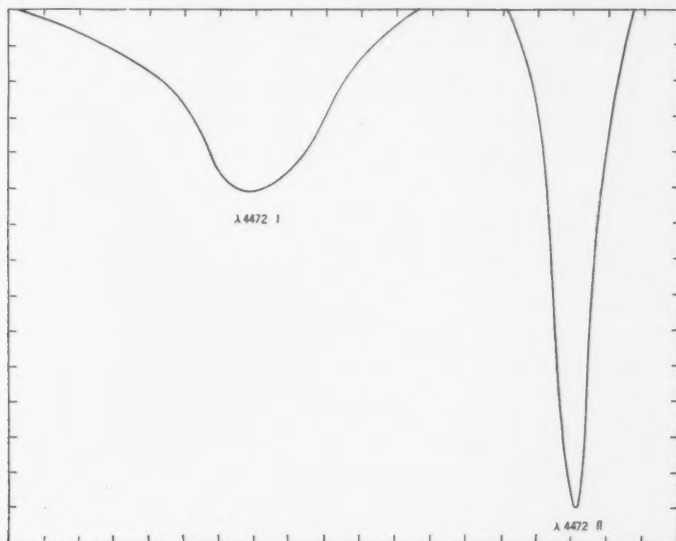
Fig. 4.—Corrected contours of the two components of α Virginis. The curves show the shapes of the lines as they would have been in the absence of overlapping with the continuous spectra. The units in the abscissa and in the ordinate are the same as in Fig. 3.

employed by Elvey. Consider the disk of the star, and imagine that the x-axis lies in the equatorial plane. If the equatorial velocity is $v \sin i$ then any point on the disk has a projected velocity

$$\frac{x}{r} \, v \cdot \sin i \, ,$$

where r is the radius. Imagine that the star is subdivided into an infinite number of sections, parallel to the axis of rotation. Each section gives an intensity $j = f(\lambda - \lambda_o)$, where

$$\lambda_o = \lambda_o' + \lambda_o' \, \frac{v \cdot x}{r} \sin i \, .$$

The area of each section is $2\sqrt{r^2-x^2}\,dx$. Multiplying j by this area and integrating over x we obtain the intensity of the light from the whole disk. To express this, as is usual, in units of the continuous spectrum, we divide the result of the integration by the integrated intensity of the continuous spectrum, which is equal to $\pi r^2 i_1$.
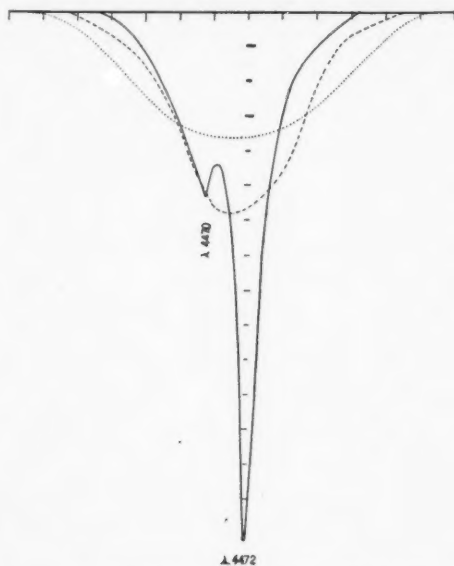
FIG. 5.—Effect of rotation upon the contour of the line λ 4472 The full line represents a non-rotating star. The two other curves correspond to equatorial velocities of 170 and 340 km/sec. The units in the abscissa and in the ordinate are the same as in Fig. 3.

Since the shape of the function f is not algebraically known we perform the integration graphically. The disk is divided into finite sections (forty in this case), each having an area of

$$a = 2\int_{x_1}^{x_2}\sqrt{r^2-x^2}\,dx = \left\{x\sqrt{r^2-x^2}+r^2\,\text{arc}\sin\frac{x}{r}\right\}_{x_1}^{x_2}.$$

Each section contributes a certain intensity $j = f(\lambda - \lambda_0)$ proportional to its area a. Rotation shifts each of these contributory contours by an amount equal to

$$\frac{x_1+x_2}{2r}\,v\cdot\sin i\,.$$

We thus have a number of contours,

$$j_n = f\left[\lambda - \left(\lambda_0' + \lambda_0' \frac{x_1 + x_2}{2r}\, v \cdot \sin i\right)\right] a_n .$$

The final curve is obtained by taking:

$$I = \frac{1}{\pi r^2 i_1} \sum j_n a_n .$$

This summation has been made for two arbitrary values of $v \sin i$, viz., 340 and 170 km/sec. (Fig. 5). It will be seen by superposition of the contour of the violet component of Figure 3 with the curves of Figure 5 that $v \sin i = 200$ km/sec. gives a good representation.

We have neglected here the effect of darkening at the limb. From the computations of Shajn and the writer it would seem that this is not important. It might perhaps tend to increase slightly the value of $v \sin i$.

The second component has almost no rotation. Since it is doubtful whether it would be possible to measure by this method a rotational velocity of less than 50 km/sec. we can only state that the rotation of the second component does not much exceed this value. If we tentatively assume $v_2 \sin i = 50$ km/sec., we obtain

$$\frac{v_1}{v_2} = \frac{r_1}{r_2} = 4 .$$

This is in good agreement with expectation. We have for the two components[1]

$$\log r_1 = \frac{5900}{T_1} - 0.2 M_1 - 0.02 , \tag{8}$$

$$\log r_2 = \frac{5900}{T_2} - 0.2 M_2 - 0.02 , \tag{9}$$

where T_1 and T_2 are the temperatures and M_1 and M_2 the visual absolute magnitudes. In this case $T_1 = T_2$ and $M_1 - M_2 = -2.4$. By subtraction:

$$\log \frac{r_1}{r_2} = 0.2(M_2 - M_1) .$$

[1] Russell, Dugan, and Stewart, *Astronomy*, 2, 738, 1927.

Consequently $r_1/r_2 = 3$, in fair agreement with the value obtained before.

While too much stress should not be placed upon the numerical values of $v \sin i$, it is clear that interesting information is contained in the contours of the components of spectroscopic binaries. For example, if the two corrected contours, j_1 and j_2, are similar, we can safely say that the diameters of the two stars are also about the same. Of course, this applies only to double stars with short periods, since it is essential that the periods of revolution and of rotation are the same. We do not know exactly at which stage this equality begins to break down. However, there is very little doubt that it is fulfilled in all binaries with periods of the order of a few days.

VI. ROTATION IN η URSAE MAJORIS

Figures 6 and 3 contain the original tracing and the contours of λ 4472 and λ 4481 for the star η Ursae Majoris. This is not a spectroscopic binary of short period and large amplitude[1] like α Virginis. The value of $v \sin i$ is again close to 200 km/sec. We are thus dealing here with a single star of the same type and presumably the same luminosity as the stronger component of α Virginis. Now this latter star is what Jeans calls a very young binary. Using equations (8) and (9) and substituting $T_1 = T_2 = 20, 000°$ and $M_1 = -2.0$, we find[2]

$$r_1 = 3 \times 10^6 \text{ km},$$

$$r_2 = 1 \times 10^6 \text{ km}.$$

From the orbit we have

$$a_1 \sin i = 7 \times 10^6 \text{ km},$$

$$a_2 \sin i = 11 \times 10^6 \text{ km}.$$

[1] It was announced as a spectroscopic binary by L. L. Mellor (*Publications of the Observatory of the University of Michigan*, **3**, 72, 1923) but the range is small and the period is probably long. None of our plates shows any indication of duplicity. Consequently the observed spectrum refers to a single star.

[2] Since $r_1 = 3 \times 10^6$ km and $P = 4.0$ days, the theoretical equatorial velocity of rotation should be $v_1 = 2\pi r_1/P = 55$ km/sec. This seems to indicate that we have used too faint a magnitude for M_1 in (8).

The inclination[1] is probably close to 90°, so that

$$a_1 + a_2 = 18 \times 10^6 \text{ km} .$$

The distance between the surfaces of the two components is only about 3.5 times the sum of their radii. Consequently it is probable that the angular rotational velocity of the stronger component of α Virginis is not far from the critical value at which a double star merges into a rapidly rotating single body. For η Ursae Majoris the
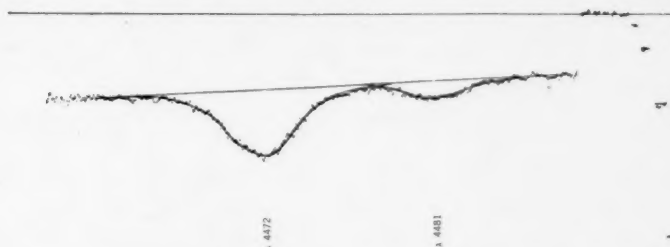
Fig. 6.—Microphotometer tracing of Plate R 1774 of η Ursae Majoris, taken on March 3, 1930, at 6^h 35^m U.T.

angular velocity is of the same order of magnitude. We conclude therefore that this star is also rather close to the critical stage where break-up may occur.[2]

VII. STABILITY

It may be of interest to compute the value of the quantity $\omega^2/2\pi\gamma\rho$, which is characteristic for the state of stability of a rotating star. Following Tissérand's method,[3] we have for the star

$$\frac{\omega_1^2}{2\pi\gamma\rho_1} = \frac{2\pi}{\gamma\rho_1 P_1^2} ,$$

[1] α Virginis was tentatively announced as an eclipsing variable by J. Stebbins (*Astrophysical Journal*, **39**, 475, 1914). However, in a later publication Stebbins did not include this star among those which are definitely known to be eclipsing variables (*Publications of the Washburn Observatory*, **15**, 56, 1928). Professor Stebbins has informed me that the question is not definitely settled, since there are few suitable comparison stars of similar spectral type in the vicinity of α Virginis.

[2] The angular velocity $\omega = 2\pi/P$ may be considered as a measure of the distance between the two components of a double star. Consequently, whatever the origin of the binary, there is a critical value of ω such that the components are just in contact. At this stage the double star ceases to exist as such and should be considered as a single body.

[3] *Traité de Mécanique Céleste*, **2**, 92, 1891.

and for the earth,

$$\frac{\omega_2^2}{2\pi\gamma\rho_2} = \frac{2\pi}{\gamma\rho_2 P_2^2} = 0.00230 \ .$$

Consequently

$$\frac{\omega_1^2}{2\pi\gamma\rho_1} = 0.00230 \left(\frac{P_2}{P_1}\right)^2 \left(\frac{\rho_2}{\rho_1}\right) \ .$$

For the earth $P_2 = 1$ day, while for a Virginis $P_2 = 4$ days. The density of the earth is 5.5 gr/cm³. Consequently

$$\frac{\omega_1^2}{2\pi\gamma\rho_1} = \frac{0.00079}{\rho_1} \ .$$

The mean density of the B-type stars ranges from about 0.01 to 0.1 gr/cm³. Using the two limits, we find

$$0.0079 \leq \frac{\omega_1^2}{2\pi\gamma\rho_1} \leq 0.079 \ .$$

If the laws deduced by Jeans and others for rotating homogeneous liquid bodies are applicable to the stars, the foregoing result would indicate that the star is stable and that the figure of equilibrium is probably a Maclaurin spheroid, the meridional cross-section of which has an eccentricity of not more than about 0.5. Since the density of a Virginis is not known, the quantity $\omega^2/2\pi\gamma\rho$ cannot be determined any closer. However, a shortening of the period to about 2.8 days, with constant dimensions and density, would bring the angular velocity rather dangerously close to the point where the Maclaurin ellipsoids lose stability. It seems quite possible that such stars exist. In fact, a Virginis is only one of many examples of rapid rotations, and it is not at all improbable that such rapidly rotating binaries as V Puppis and μ^1 Scorpii approach the critical point even closer than a Virginis.

YERKES OBSERVATORY
    May 22, 1930

EVIDENCE FROM BAND SPECTRA OF THE EXISTENCE OF A CARBON ISOTOPE OF MASS 13

By ARTHUR S. KING and RAYMOND T. BIRGE[1]

ABSTRACT

Isotope C^{13}.—Evidence of the existence of this isotope *appeared first in a faint band structure* with head at λ 4744.5, showing on electric-furnace spectrograms in which the Swan (C_2) band λ 4737 was of high intensity. The structure of the latter band being known, the *displacements of the head and 15 component lines* of the faint band *were measured* from corresponding lines of the primary band. An application of the *quantum theory* of band spectra then *showed* that the *band* λ 4744.5 *was due to a molecule* $C^{13}C^{12}$, *the emitter* of λ 4737 being the molecule $C^{12}C^{12}$.

Further experimental work, together with examination of older material, showed that C^{13} *enters into molecules producing isotopic bands* connected with the Swan band λ 4382, the cyanogen bands λ 3883 and λ 3590, and the carbon monoxide bands between λ 1200 and λ 1500.

The *isotopic shift* for the (1,0) band λ 4737 has been *studied in detail* as far as the observational material permitted. The *staggering* of the main band, the presence of *perturbations*, and the *methods of computing* the *vibrational* and *rotational shifts* are discussed. The isotopic shift for this band is found to be about 7.6 A, and the algebraic mean of the differences O−C is −0.0096 A, or 1 part in 800, a difference within the errors of measurement. It follows from this that the *mass ratio* of the two carbon isotopes is *12 to 13, accurate to about 1 part in 10,000.*

Other isotopic bands.—A faint band structure to the red of the (2,0) band at λ 4382 has a head with an isotopic shift of 12.58 A, agreeing closely with the calculated shift of 12.57 A.

Evidence of a $C^{13}N^{14}$ molecule *is found in the cyanogen band* λ *3883*, where *six faint lines* evidently belong to an isotopic band, their *measured shifts* from corresponding primary lines differing from the calculated by an average amount +0.0057 A. This small discrepancy may be due to an electronic isotope effect. *The (1,0) band of this system,* λ *3590, also shows an isotopic band in the furnace spectrum,* the observed shift from the primary head being 5.3 A, agreeing with the calculated shift of 5.30 A.

These results incidentally furnish quantitative evidence that the *emitters of the Swan and "cyanogen" bands are the C_2 and CN molecules,* respectively.

The *relative abundance* of C^{13} and C^{12}, estimated very roughly from photographic intensities of the bands, *appears to be of the order 1 : 400.* Such estimates are rendered uncertain, however, by an apparent dependence on the method of excitation. The furnace and other *low-excitation sources, notably N-type stars,* are *very effective in emitting the C^{13} spectrum.* In the arc, however, the *only evidence* of C^{13} thus far is the group of lines in the cyanogen band λ 3883, this band being emitted with extraordinary intensity by the arc. *Conditions* in the *three typical sources*—arc, furnace, and star—are discussed with regard to their possible bearing on the excitation of bands in which C^{13} is concerned.

INTRODUCTION

Practically all of our knowledge concerning the isotopes of non-radioactive elements is due to the work of Aston and Dempster, who used some form of mass spectrograph. The most recent list[2]

19

of such isotopes gives fifty-seven elements that have been examined for isotopes, of which twenty-three are apparently single, so far as evidence from the mass spectrograph is concerned. Among this list of twenty-three are the two elements, oxygen and carbon, which from their integral atomic weights and other evidence would seem least likely to consist of two or more isotopes.

Under such circumstances, the discovery by Giauque and Johnston[1] of isotopes of oxygen, of mass 17 and 18, is of great significance. The relative abundance of these two new atomic species is very small, compared to the more usual O^{16} species, and this explains the failure of the mass spectrograph to detect them. Thus O^{18} has the same mass as H_2O, except for a slight difference of packing effect,[2] and it is difficult to exclude all moisture from such an apparatus. On the other hand, the spectrum of the O_2 molecule, as employed by Giauque and Johnston, is eminently suitable for the purpose.

The initial application of the quantum theory of band spectra to the matter of isotopes is due independently to Loomis[3] and Kratzer.[4] Later work by Mulliken and others[5] has resulted in the identification in band spectra of a number of isotopes already known from the mass-spectrograph work. This spectroscopic work was of special importance in that it constituted perhaps the most direct and convincing confirmation of the present accepted interpretation of molecular spectra. As a result of it, one can now predict with great confidence just how the spectrum of a molecule containing any atomic species, such as O^{17} or O^{18}, should differ from the spectrum of the molecule containing only the more usual species (here O^{16}) of the same element. Within reasonable limits, the relative scarcity of an atomic species is no bar to its successful identification, for by sufficiently long exposures the new species can be made evident.

[1] *Journal of the American Chemical Society*, **51**, 1436 and 3528, 1929.

[2] At the present time it would appear that the mass of O^{18} is close to 18.000, while that of H_2O is close to 18.014. Hence Aston should be able to distinguish between the two in his precision mass spectrograph, described in 1927 (*Proceedings of the Royal Society*, A, 115, 487, 1927).

[3] *Nature*, **106**, 179, 1920; *Astrophysical Journal*, **52**, 248, 1920.

[4] *Zeitschrift für Physik*, **3**, 460, 1920; **4**, 476, 1921.

[5] See F. W. Loomis, "Molecular Spectra in Gases," *Bulletin of the National Research Council*, **11**, Pt. 3 (No. 57), chap. v, 1926.

Presence of impurities, like H_2O, is irrelevant, since the spectrum of any molecule, such as O_2, is unique. The spectra of molecules containing different atomic species, such as $O^{16}O^{16}$ and $O^{16}O^{18}$, differ, to a first rough approximation, merely in a "shift" of each band to higher or lower frequency, the direction and extent of the shift (vibrational-isotope effect) depending on the masses concerned and on the quantum designation of the particular band. A more extended and correct description of the isotope shift is given later in this paper.

Giauque and Johnston, in their work on the oxygen isotopes, used spectroscopic data obtained by H. D. Babcock for atmospheric lines in the solar spectrum, taken at periods of very low sun when the atmospheric path was of maximum length. These oxygen isotope lines have not yet been detected in laboratory sources.

In the course of investigations by one of the writers on the electric-furnace spectra of metals, spectrograms were occasionally obtained in which high temperature and long exposure caused the Swan spectrum of carbon, from the material of the graphite tube, to appear with high intensity. The origin of this band system has long been in dispute, but recent evidence[1] has pointed rather clearly to the diatomic molecule C_2. The present paper furnishes definite evidence of the correctness of this conclusion.

On the electric-furnace plates just mentioned, a faint band appeared with head at λ 4744.5, the structure being visible until it merged into the head of the strong (1,0) band of the Swan system at λ 4737. Although not accounted for by the analysis of the various series of the λ 4737 band, a physical connection with the latter seemed very probable, borne out by the fact that in spectra of N-type stars, the so-called "carbon stars," the strong band at λ 4737 is accompanied by a fainter one whose position agrees closely with that in the furnace spectrum. This faint band has also figured in the literature, although its band character seems not to have been recognized. Thus Raffety[2] speaks of a well-marked single "line" at λ 4743, associated with the Swan bands, and in Figure 14, Plate XIII, accompanying his article, this line appears clearly, and has

[1] W. E. Pretty, *Proceedings of the Physical Society of London*, **40**, 71, 1928.

[2] *Philosophical Magazine* (6), **32**, 546, 1916.

in fact more the appearance of a band. It appears also in Figure 1, Plate 4, of an article by Johnson[1] on the Swan bands, although it is not mentioned in the article.

Inspired by the discovery of the isotopes of oxygen, the writers examined this faint band at λ 4744 in connection with a possible isotope of carbon. In spite of the faintness of the band, the head and six lines could be recognized and measured. The six corresponding lines in the main band, with head at λ 4737, were obvious, and we therefore had six measurable "shifts." The quantum analysis of the Swan band system was known from the work of Shea,[2] and calculations showed immediately that the λ 4744 band agreed quantitatively (to a fraction of 1 per cent) with the molecule $C^{13}C^{12}$ as an assumed source, the main band being due to $C^{12}C^{12}$. The announcement[3] of this was followed in a few days by another, in which one of the writers[4] showed that certain faint lines found by one of us[5] in electric-furnace spectra, scattered among the component lines of the λ 3883 band of cyanogen, agreed quantitatively with the molecule $C^{13}N^{14}$ as an assumed source, and that a series of subheads, found by Hopfield and Birge[6] in the absorption spectrum of carbon monoxide, agreed with $C^{13}O^{16}$ as an assumed source.

Systematic experimental work was then undertaken, in order to bring out the structure of the band at λ 4744 as fully as possible, to look for evidences of C^{13} in other bands, and to determine if possible the conditions governing the appearance of λ 4744, which in the laboratory occurred only in the spectrum of the furnace. As a result of this work, the λ 4744 band has been obtained in both second- and first-order furnace spectra, taken with the 15-foot concave grating. Its strength in the first order is now sufficient to enable at least fifteen lines to be measured. The corresponding isotope band accompanying the (2,0) band at λ 4382 has been identified, although its position and extreme faintness make accurate measure-

[1] *Philosophical Transactions of the Royal Society of London*, A, **226**, 157, 1927.

[2] *Physical Review*, **30**, 825, 1927.

[3] *Ibid.*, **34**, 376, 1929; *Nature*, **124**, 127, 1929 (dated June 24).

[4] Birge, *Physical Review*, **34**, 379, 1929; *Nature*, **124**, 182, 1929 (dated June 29).

[5] King, *Mt. Wilson Contr.*, No. 194; *Astrophysical Journal*, **53**, 161, 1921.

[6] *Physical Review*, **29**, 922, 1927.

ments very difficult. In cyanogen we have found the $C^{13}N^{14}$ band accompanying the (1,0) band at λ 3590.

Measurements on all of the new bands have been made by each of the writers, and certain preliminary conclusions have been reached and announced.[1] The work already finished is sufficient to establish without question the existence of an isotope of carbon, of mass 13. We are, however, trying to get quantitative measurements of the isotope shift, from which one may obtain an extremely accurate evaluation of the mass ratio and "packing effect" of the two atomic species.[2] We are also investigating the relative intensity of the "isotope" bands, as compared with the main bands. This relative intensity seems, strangely enough, to be a function of excitation conditions. Both of these problems have proved much more difficult than anticipated, and are far from solved at the present time. In regard to the first, it appears necessary to make new measurements, and from these to derive new constants for all the bands under consideration. Such work must necessarily consume a considerable period of time.

The purpose of the present article is then to present, in a semiquantitative manner, the material bearing on the existence and appearance of the carbon isotope of mass 13. Further details bearing on various aspects of this matter will be published in a series of later articles.

THE $C^{13}C^{12}$ BAND AT λ 4744.5

The (1,0) band of the Swan system is λ 4737, which represents a $^3\Pi - {}^3\Pi$ electronic transition in the $C^{12}C^{12}$ molecule. It corresponds directly to the B band of atmospheric O_2 in the solar spectrum, and is the best available band for observing the C^{13} isotope and for measuring the isotope shift. For this reason special efforts have been made to obtain the $C^{13}C^{12}$ isotope band at λ 4744.5 with as great intensity as possible.

The regular furnace tubes of highly purified Acheson graphite, 19 mm in outside diameter and 12.5-mm bore, the heated portion being 20 cm long, were used in the investigation. As the Swan bands appear in this furnace at about 2400° C and as a high intensity of

[1] *Ibid.*, **35**, 133, 1930 (abstract 3).
[2] See W. F. Giauque, *Nature*, **124**, 265, 1929.

λ 4737 was required, tube temperatures of 2800° to 2900° C were employed for the best results. The use of high-contrast plates gave an improved resolution of the band structure. Details of the isotope band at λ 4744 were brought out best by the first order of the 15-foot concave-grating spectrograph, on account of the negligible strength of the ghosts in this order. Numerous spectrograms were also made in the second order, to obtain superior resolution near the λ 4737 head, but the isotope band was somewhat blurred by a ghost at about λ 4740.

Plate I is an enlargement of our best first-order spectrogram and shows the heads of the main (1,0) and (2,1) bands, at λ 4737 and λ 4715, respectively, and also the new isotope band at λ 4744.5. The structure just at the λ 4744 head resembles that of the heads of the main Swan bands, although the isotope band as a whole is by no means a copy of its primary. The unresolved head λ 4744.5 has a sharp companion line at λ 4744.63, while on the violet side fifteen band lines are measurable on the best plates before the isotope band merges in the primary head. The identity of the various lines in both the main band and the isotope is shown on the photometric curve a of Plate II, which, as in the case of other such curves shown here, was traced with the Zeiss microphotometer in the physics department of the University of California. Curve a was made from the first-order spectrogram shown in Plate I. Curve b covers the same region, using a second-order plate. As just noted, some extraneous structure appears in the second-order isotope lines.

Details concerning the fine structure of the Swan bands may be found in the articles by Shea[1] and Johnson.[2] Page 173 of the latter article shows a drawing of the fine structure near the head of λ 4737, and Plate 4 of the same article reproduces photographs of this band, in vacuum tubes and in arc sources. Our own furnace spectrograms correspond to a temperature intermediate between the tube and furnace, but show some striking differences in intensity distribution which will be discussed in later articles. We are here interested only in the general features of the fine structure. Each band consists of P and R branches only. Each line is itself triple, the spacing being greatest near the origin of the band and diminishing rapidly as the

[1] Loc. cit.   [2] Loc. cit.

PLATE I
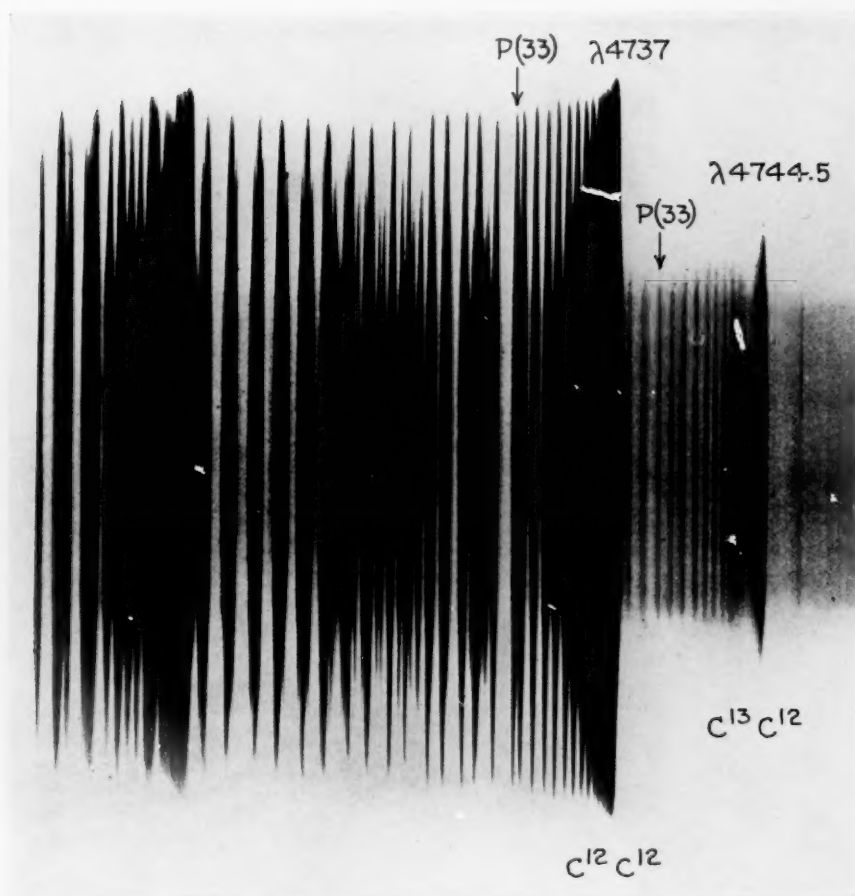
THE CARBON BAND λ 4737 AND ISOTOPE BAND λ 4744.5

PLATE II
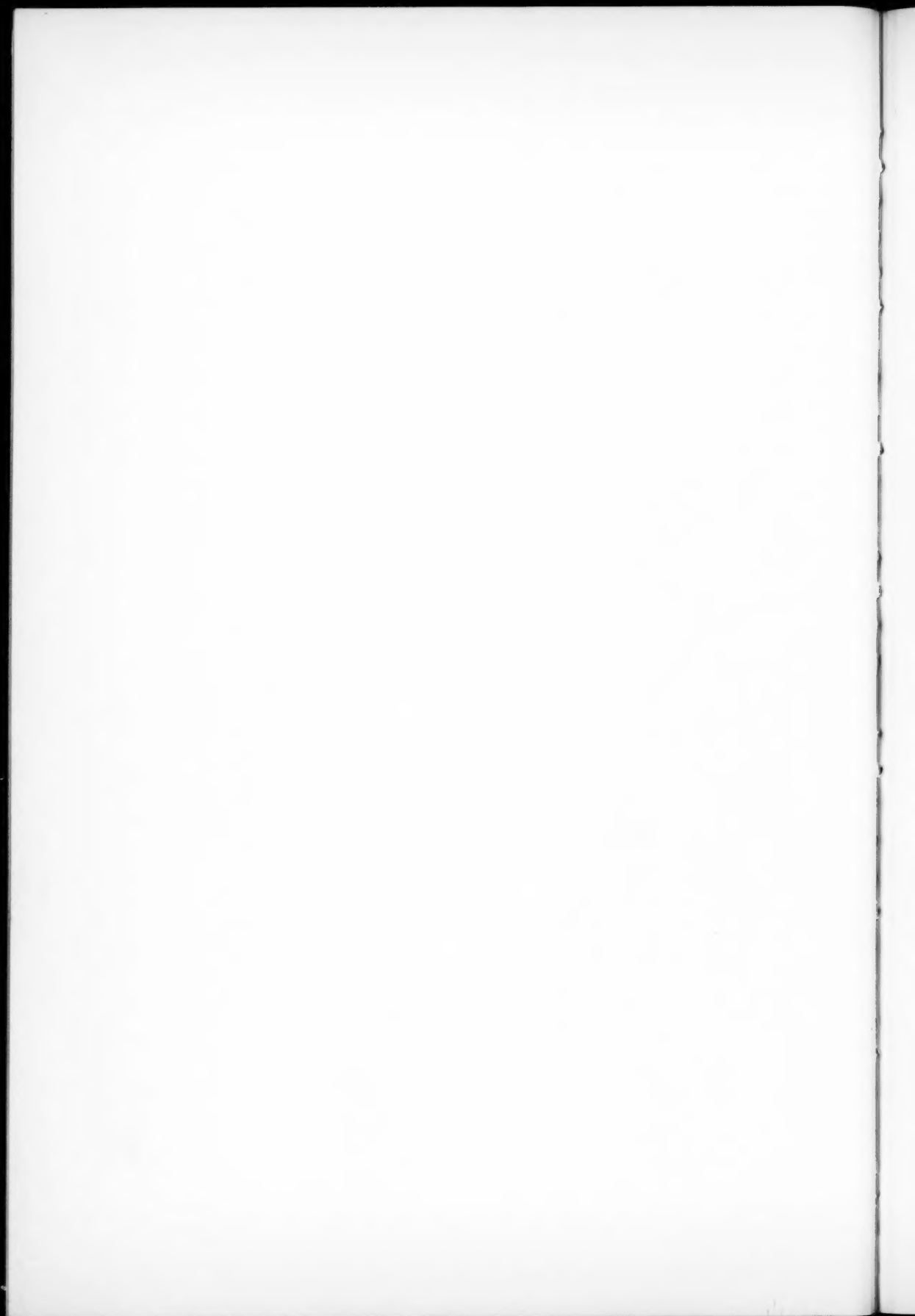
4737
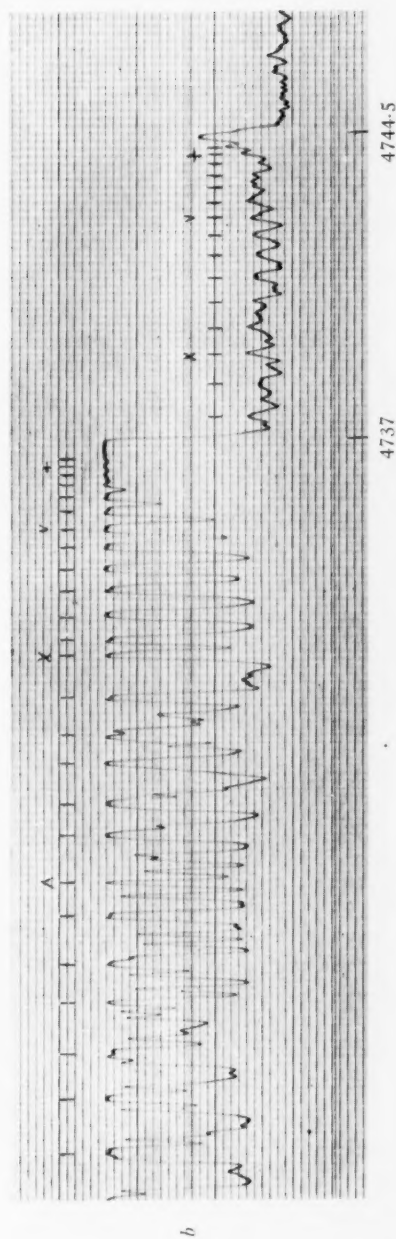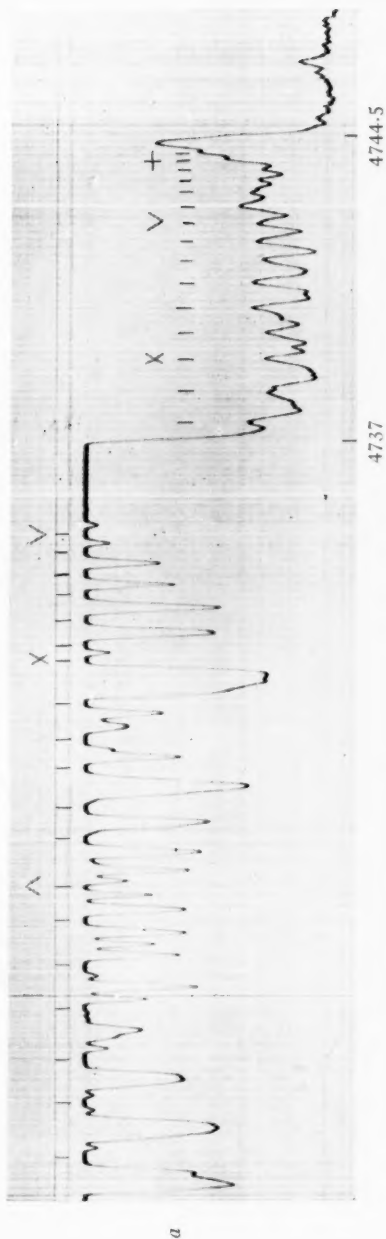
4744·5

4737

4744·5

MICROPHOTOMETER CURVES OF THE CARBON BAND λ 4737 AND ITS
ISOTOPE λ 4744·5

a) Curve from first-order spectrogram
b) Curve from second-order spectrogram
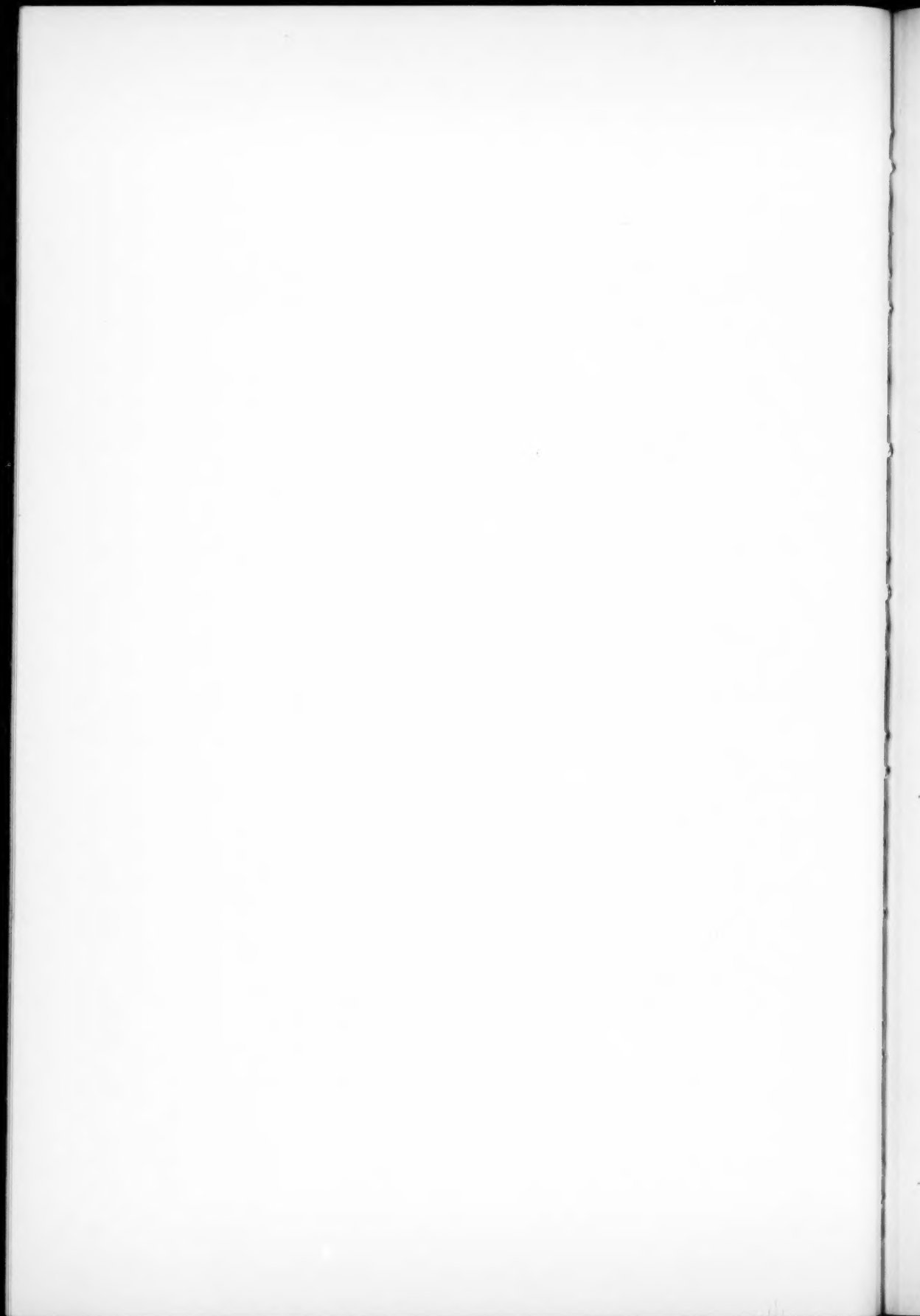
rotational quantum number J increases.[1] The calculated origin v_0 of the (1,0) main band is at λ 4730.938 (21,131.56 cm^{-1} in vacuum),[2] while that of the isotope band should be at about λ 4738.72 (21,-096.85 cm^{-1} in vacuum). From the origin the P branch runs toward the head, then reverses and returns across the origin. In the spectrograms here used, only the returning P branch appears. Both Johnson and Shea use the same numerical labels for the various triplets composing each branch, and for convenience their labels are retained. Without going into the rather complex details of nomenclature,[3] it may be noted that in a line such as $P(30)$, the number 30 is what Shea assumed the actual rotational quantum number J'' of the lower state to be, for the central line of the three composing a "natural" triplet.[4]

Calculations show that the head of the λ 4737 band occurs at P (17). In the isotope band the first line that can be measured is $P(21)$, and the last is $P(35)$. This gives fifteen lines for measuring the shift, although only the lines $P(25)$ to $P(35)$ have as yet been used. These various lines are indicated in both the main and isotope bands, in Plate II, curves a and b. Line $P(22)$ is labeled by a plus sign ($+$), P (27) by a v, $P(33)$ by an x, and $P(39)$ (main band only) by an inverted v. The multiple character of the lines of the main band is shown plainly in curve b. The isotope lines appear, however, merely as broad, single lines. This is as expected. It is well known that successive triplets in the Swan bands are "staggered" (i.e., alternate lines displaced from a smooth curve), a phenomenon connected with the fact that, theoretically, a $^3\Pi$ level has a sixfold multiplicity. The doubling of each of the three levels is

[1] See Fig. 3, p. 841, of Shea, *loc. cit.*

[2] This is a new calculation and differs very slightly from Shea's value. Johnson's values of v_0, for the various bands, are incorrect, due to an error in his assignment of J-values. See Shea, *loc. cit.*

[3] We are following, so far as convenient, the new nomenclature proposed by R. S. Mulliken, in an article now in press. See also R. S. Mulliken, *Reviews of Modern Physics*, **2**, 60, 1930.

[4] It is actually the value of $J''+1$, according to the new mechanics, and is also the value of $K''+1$ for all three lines of the triplet. We shall be concerned with the rotational energy change, corresponding to each line, and this is given to the first approximation by $B'(K'+1/2)^2 - B''(K''+1/2)^2$. Thus, in calculating the rotational energy change for $P(30)$, one would write $B'(28.5)^2 - B''(29.5)^2$.

termed "Λ-type doubling." In a symmetrical molecule, composed of atoms having no nuclear spin moment, like $C^{12}C^{12}$, half of the levels are absent, the remaining levels constituting alternately the upper and lower components of the Λ-type doubling. This produces the staggering of the lines. The isotope molecule, $C^{13}C^{12}$, is, however, not symmetrical and all six levels should occur. The presence of twice as many levels, in an unsymmetrical isotope molecule, has been proved[1] in the case of O_2. Hence each line of the $C^{13}C^{12}$ band is presumably an unresolved sextet, and successive lines should show no staggering.

Another noteworthy difference between the main and isotope bands concerns the location of perturbations. The line $P(33)$ occurs far to the red of its expected position in the main band, but shows no irregularity in the isotope band. Such perturbations are now explained as a "resonance" phenomenon, due to the near-coincidence of a level, or levels, of one electronic state with those of another electronic state.[2] Since all of the energy-levels of the isotope molecule $C^{13}C^{12}$ are shifted with respect to those of the primary molecule, and shifted by varying amounts, one should indeed expect perturbations in the isotope molecule, but not at the same value of J. There is no large perturbation in the isotope molecule within the short range covered by the observational data.

As already noted, we have at this time no final quantitative results to report. The following data and discussion will indicate merely in a general way the nature of the evidence. Table I, second column, gives the average of a number of sets of readings for the center of gravity of each triplet in the main band, as well as for the sharp line at the red edge of the head. The third column gives similar data for the isotope band. The absolute scale of wavelengths is based on a direct comparison with iron normals. Because of the great intensity difference between the λ 4737 and λ 4744 bands, they had to be measured on different plates. As a comparison standard, we used the line $P(39)$ at λ 4726.316 in the primary band, since this is sharp and apparently single on all plates. The observed isotope shift is given in the fourth column of Table I,

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[1] Giauque and Johnston, *loc. cit.*

[2] See, for instance, G. H. Dieke, *Nature*, **123**, 446, 1929.

and the calculated in the fifth column. The calculated shift was obtained as follows.

The isotope shift[1] is composed of three parts: electronic, vibrational, and rotational. It is to be anticipated that the electronic shift is small (0–0.05 cm^{-1}). This effect can be measured and eliminated only by means of observations on two different bands having a common vibrational state. At present we have usable data only

TABLE I

	λ_{air} $C^{12}C^{12}$	λ_{air} $C^{13}C^{12}$	Obs. Shift	Calc. Shift
			A	A
$P(35)$............	4729.913	4737.672	7.759	7.812
$P(34)$............	4730.803	4738.430	7.627	7.666
$P(33)$............	4731.833	4739.179	7.346	7.353
$P(32)$............	4732.210	4739.837	7.627	7.645
$P(31)$............	4732.830	4740.452	7.62 :	7.659
$P(30)$............	4733.439	4741.068	7.629	7.618
$P(29)$............	4733.954	4741.605	7.651	7.645
$P(28)$............	4734.481	4742.072	7.591	7.594
$P(27)$............	4734.921	4742.499	7.578	7.601
$P(26)$............	4735.352	4742.906	7.554	7.564
$P(25)$............	4735.697	4743.278	7.581	7.586
$P(24)$............	4735.979	4743.568	7.589	....
$P(23)$............	4736.238	4743.820	7.582	....
$P(22)$............	4736.456	4744.037	7.581	....
$P(21)$............	4736.624	4744.221	7.597	....
Head............	4737.059	4744.628	7.569	7.55 .

for one band, and therefore cannot evaluate this shift, if present at all. The main portion of the observed shift is vibrational, and the magnitude of a vibrational shift varies roughly with the change in the vibrational quantum number v. For this reason the calculated shift in the $(0,0)$ band is very small,[2] while that in the $(2,0)$ band is nearly double the shift in the $(1,0)$ band. The vibrational shift applies strictly to the origin of a band, and to calculate it one needs the constants giving the origins of the bands of a system as a function of the vibrational quantum numbers v' and v''. For the $(0,0)$ band we have calculated the origin directly from the data given

[1] See R. T. Birge, *Transactions of the Faraday Society*, **25**, 718, 1929, for a brief review of the theory, and particularly of the methods used in the present work.

[2] The $(0,0)$ band at λ 5165 is photographically so much fainter than the $(1,0)$ band that the isotope is apparently not detectable with the material now available, regardless of the magnitude of the shift.

by Johnson, the result[1] being 19,377.71 cm^{-1}. With the exception of the origins of the (0,0) and (1,0) bands, we have used the vibrational constants determined by Johnson.[2] Fuller details will be given in later papers.[3]

The finally adopted vibrational constants are

$$\omega'_e = 1792.55 \text{ cm}^{-1} \qquad \omega'_e x'_e = 19.35 \text{ cm}^{-1}$$
$$\omega''_e = 1641.55 \text{ cm}^{-1} \qquad \omega''_e x''_e = 11.67 \text{ cm}^{-1}$$

It is known, from the work of Johnson, that the $\omega_v : v$ curve is practically linear, for both upper and lower electronic levels, and the vibrational shift is then given to all desired accuracy, by

$$\nu_2 - \nu_1 = (\rho - 1)(\omega'_e u' - \omega''_e u'') - (\rho^2 - 1)(\omega'_e x'_e u'^2 - \omega''_e x''_e u''^2) . \qquad (1)$$

In this equation $u = v + 1/2$, and is that number which is zero when the vibrational energy is zero. It may be called the "effective" vibrational quantum number. For the λ 4737 band, $u' = 3/2$, $u'' = 1/2$, and from equation (1), $\nu_2 - \nu_1 = -34.713$ cm^{-1}, with a probable error of about 0.01 cm^{-1}.

For the rotational shift we used the method already proposed by one of us.[4] The rotational constants employed are those given by Shea.[5] These are not complete for the $v' = 1$ state, and for that reason the calculated rotational energy is not in particularly good agreement with the observed value. The method already described gives, however, quite reliable results, even in the case of such discrepancies, and the calculated values are therefore trustworthy so far as these facts are concerned.

The real difficulty in calculating the rotational shift arises from

[1] Shea's value, 19,379.20, is incorrect, due to some arithmetic error.

[2] *Loc. cit.*

[3] As has been noted, we intend to obtain new constants for this system, and hence those now being used are to be considered as provisional. It should be remarked, however, that a change in the constants affects the calculated shifts very slightly. Thus a change of the origin will, in the first approximation, merely redistribute the energy in such a way that a change of 1 cm^{-1} in the actual frequency of the origin will change the calculated shift by only 0.02 cm^{-1}.

[4] Birge, *Transactions of the Faraday Society*, **25**, 718, 1929.

[5] *Loc. cit.*

quite different sources. In the first place, the isotope lines should not be staggered. We have therefore calculated smoothed positions for the lines of the main band (i.e., with the staggering eliminated), and then have calculated theoretical shifts with respect to these smoothed positions. The shifts were then corrected for the observed staggering. The perturbation at $P(33)$ was similarly treated. Hence the values in the fifth column of Table I give the calculated shifts from the observed lines of the main band to a set of lines of the isotope band assumed free from staggering and from perturbations. Now the observed data for the isotope band give no indication of staggering, but in all probability perturbations are present somewhere, and it is now known that a large perturbation at one line affects the positions of all lines in the vicinity.[1] Moreover, new measurements of the main band indicate unmistakably such an effect in the vicinity of $P(33)$, and this has not been taken into consideration in calculating the shifts. In other words, all lines of both the main and the isotope bands are, in all probability, irregular in position in a systematic manner connected with the larger perturbations, and it is very difficult in the calculations to eliminate the effect of such general irregularities. The poor agreement in the case of lines $P(34)$ and $P(35)$ in Table I is undoubtedly due to this cause. Without going into further details concerning this very involved matter, it may be said that, with sufficient experimental material, one should be able to calculate shifts which, taken as a whole, are correct, but which may be quite incorrect for the individual lines. In the present case it has seemed justifiable to use only the nine lines $P(33)$ to $P(25)$, inclusive. Further measurements are needed in order to evaluate properly the shifts for $P(21)$ to $P(24)$.

For the nine lines just cited the algebraic average of the $O-C$ shifts is -0.0096 A. In other words, the observed shift is less than the calculated, on the average, by 1 part in 800. At the present time this discrepancy represents approximately the probable error and so cannot be considered significant. Now it may easily be shown that if the observed vibrational shift (in this case practically the entire shift) differs from the calculated shift by the proportional

[1] See Fig. 6, p. 90, of "Molecular Spectra in Gases," *op. cit.*, for the first known example of this, and F. A. Jenkins, *Physical Review*, **31**, 539, 1928, for more recent work.

amount Δ_s, this discrepancy may be removed by changing the mass ratio of the two atoms concerned by

$$\Delta_m = \frac{4\rho_0(\rho_0 - 1)\Delta_s}{1 - 2\rho_0^2} , \qquad (2)$$

where ρ_0 is the square root of the ratio of the assumed reduced masses of the two molecules. This formula is true only for small values of Δ_m and Δ_s. For $C^{12}C^{12}$ and $C^{13}C^{12}$, $\rho_0 = 0.98058$ and $\Delta_m = \Delta_s/12.1$. If, therefore, $\Delta_s = 1/800$, $\Delta_m = 1/9700$. In other words, the mass ratio of the two atoms is to be taken greater than $12:13$ by 1 part in 9700. As just noted, this discrepancy is too close to the experimental error to be significant, and we may therefore conclude that the two carbon isotopes have a mass ratio of 12 to 13, accurate to about 1 part in 10,000. This conclusion ignores a possible electronic shift, but such a shift, even if of magnitude 0.04 cm^{-1}, would change the mass ratio only 1 part in 10,000. Work now in progress on the oxygen isotopes, as well as previous line-spectra work, indicates that the electronic shift is not likely to exceed this amount.

The presence or absence of staggering in the isotope band concerns only the relative wave-lengths in this band, and can therefore be determined more easily and accurately than the isotope shift itself. Band series, over a short interval, are given very closely by a parabolic formula, and the second differences are therefore constant. If, however, staggering occurs, the individual second differences are alternately larger and smaller than the average. Such a test has been made on each set of measurements of the λ 4744 band. The results show that in the final average, as well as in the individual sets, there are almost exactly equal numbers of coincidences (second differences larger than the average when so predicted by staggering, etc.) and non-coincidences, and a study of the O−C values, in the case of both coincidences and non-coincidences, was equally conclusive. In other words, a second difference predicted to be larger than the average (on the basis of staggering) was just as likely to be found smaller, and by just as great an amount. It therefore seems safe to conclude that staggering is either entirely absent from the isotope band, or at most is only a small fraction of that found in the main band.

PLATE III
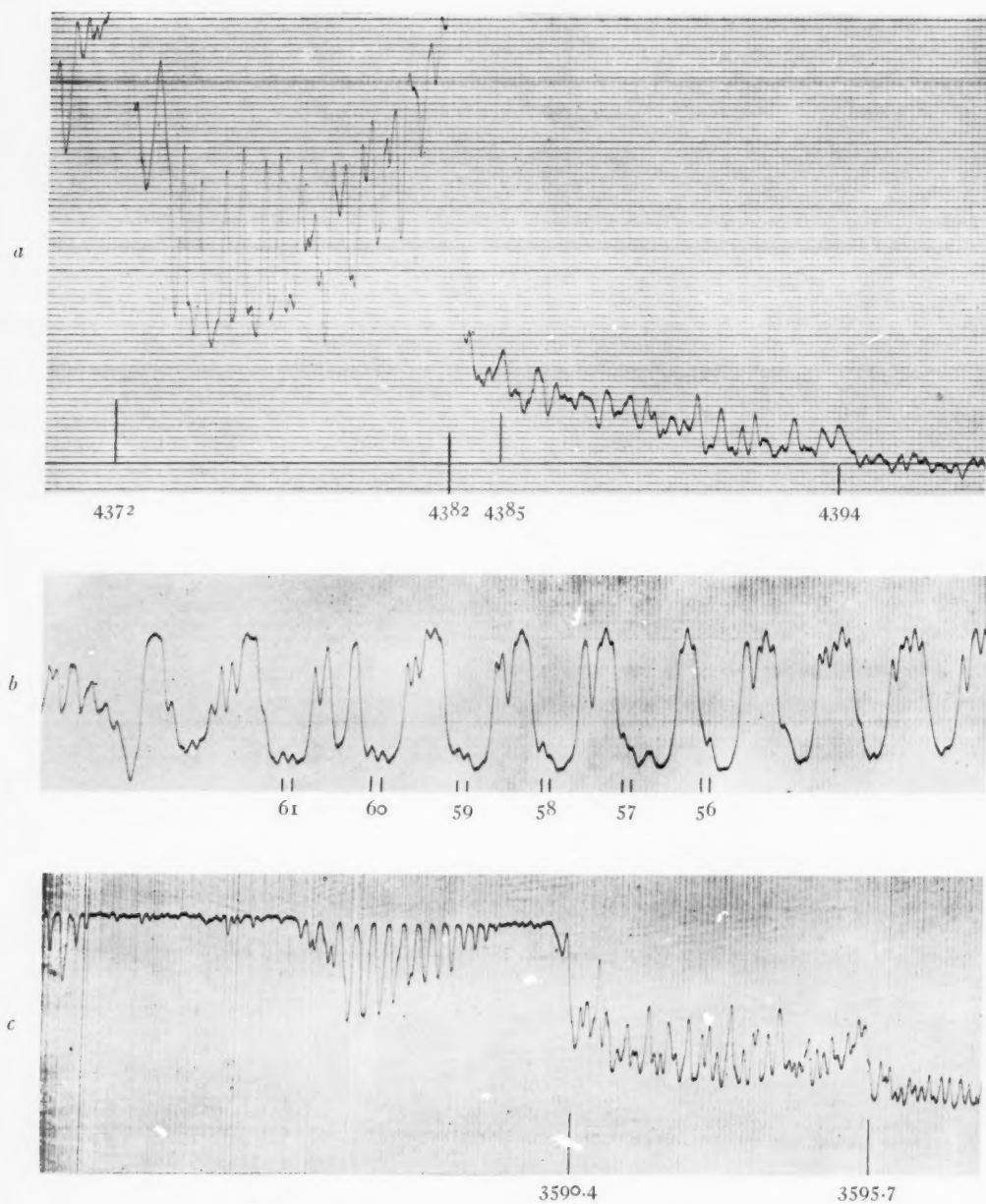
MICROPHOTOMETER CURVES OF CARBON AND CYANOGEN
BANDS AND THEIR ISOTOPES

a) λ 4382 and isotope λ 4394
b) Structure in cyanogen band λ 3883
   Marked lines belong to isotope band
c) λ 3590.4 and isotope λ 3595.7

OTHER NEW BANDS

Considerable work has been done on the remaining new bands, but the quantitative conclusions are less definite than in the case of the λ 4744 band. Plate III, curve *a*, is a photometric trace showing the (2,0) isotope band at λ 4394.5 and the main band at λ 4382.1. This curve has been made from a first-order spectrogram, with the furnace at a temperature of 3000° C. It has been very difficult to get this band at all, and even on our best plate it differs in intensity so little from the general background that it is not possible to reproduce the spectrogram. In order to bring out the isotope band properly the photometer was made very sensitive— far too sensitive for the main band. The curve shows also the (3,1) head at λ 4372.4. The corresponding isotope head should be at about λ 4385.1, and thus falls to the red of the head of the main (2,0) band. The intensity increases suddenly at this point, and we believe therefore that the isotope band is present, although this is by no means certain as yet. The two main band heads and the two isotope heads are marked on the curve.

Measurements have been made covering the region λ 4394 to λ 4372, but the isotope band is so overrun with lines of the (1,0) band, as is proved by the presence of such lines to the red of λ 4394, that analysis is difficult. Equation (1) gives a calculated vibrational shift of -66.54 cm^{-1} or $+12.812$ A at the origin of the (2,0) band. The distance from the head to the origin is about 33.2 cm^{-1}, and using for the rotational shift the approximate formula $33.2(\rho^2-1)$, one obtains for the total calculated shift at the head -65.27 cm^{-1} or $+12.57$ A. It is difficult to measure the isotope head, since no single sharp line can be seen, but our measurements give, by chance, just 12.58 A as the interval from the strong lines at the head of the main band to what appears to be the corresponding portion of the head of the isotope band. Fifteen other lines, or blends, have been measured in the (2,0) isotope band; but of these only five fall close to predicted positions. It is obvious that practically all of the lines are more or less seriously blended and therefore useless for an accurate test.

In the case of the (0,0) λ 5165 band, the calculated shift at the origin ($19,377.71$ cm^{-1} or 5159.135 A) is only -1.392 cm$^{-1} = +0.371$

A. At the head it is considerably less, namely, -0.515 cm^{-1} or $+0.137$ A. The isotope head should therefore fall just outside the main head, but so close as to be almost impossible of detection, especially when the necessary intensity of the main band is considered. It seems more likely that isotope lines could be picked up in the region lying just to the red of the head of the (1,1) band at λ 5129, since the calculated shift in this region is about 1.5 A. Unfortunately this is the most difficult region in the entire visible spectrum in which to obtain high photographic intensity.

The C^{13} isotope has, however, been detected in the corresponding region of the (0,0) band of CN (λ 3883), as has already been reported.[1]

TABLE II

K''	Main Band	Calc. Shift	Obs. Shift	O − C
	A	A	A	A
56	3875.342	+0.1317	+0.1350	+0.0033
57	3874.759	.1560	.1634	.0074
58	3874.156	.1810	.1885	.0075
59	3873.534	.2068	.2116	.0048
60	3872.891	.2335	.2451	.0116
61	3872.227	+0.2608	+0.2703	+0.0095

For this work we are using material obtained a number of years ago.[2] As in the case of the Swan bands, the $C^{13}N^{14}$ lines appear in a spectrogram taken with the electric furnace, the temperature being $2700°$ C. The lines that can be measured with any accuracy consist of six doublets lying in the region $\lambda\lambda$ 3872–3876. The appearance of the main band in this region is shown on a reproduction[3] of a very similar furnace plate taken at $2500°$ C. The isotope lines do not appear on that plate. They are shown, however, in curve b, Plate III, of the present article. This curve was made from the $2700°$ spectrogram, a third-order exposure taken with the 30-foot plane grating of the Mount Wilson Observatory. The six doublets are numbered on curve b to correspond with the numbering in the first column of Table II.

[1] Birge, *Physical Review*, **34**, 379, 1929; *Nature*, **124**, 182, 1929.

[2] King, *Mt. Wilson Contr.*, No. 194; *Astrophysical Journal*, **53**, 161, 1921.

[3] *Mt. Wilson Contr.*, No. 194, Pl. YII., *Astrophysical Journal*, **53**, Pl. II (facing p. 162), 1921.

In this case the true values of K'' are used, and the rotational energy change for each doublet is given to the first approximation by $B'(K'+1/2)^2 - B''(K''+1/2)^2$. Since the observed doublets belong to the returning P branch, $K' = K'' - 1$. The second column of Table II gives the mean wave-length of each doublet in the main band. These are smoothed values, based on the data of Uhler and Patterson.[1] Since there are no standard lines on our furnace spectrogram, we have measured only the isotope shift. Each isotope doublet lies just to the red of the corresponding main-band doublet. These main-band doublets are all self-reversed, and so appear as close triplets. They include the well-known perturbation $P(60)$, in which the doublet spacing is 0.22 A, instead of the expected 0.068 A, and in which the center of gravity is at λ 3872.855, or 0.036 A to the violet of its expected position. As in the similar situation in the λ 4737 Swan band, this perturbation does not occur in the isotope band, but its presence in the main band makes it very difficult to calculate a reliable value of the isotope shift. The large values of K'', and hence of the rotational energy, introduce another difficulty, for the higher terms of the rotational energy function are becoming appreciable, and it is necessary to have very accurate values of all the rotational energy constants. In this case we have used constants determined in some unpublished work by R. T. Birge and W. O. Smith. For the vibrational constants we have used Kratzer's values.[2] The third and fourth columns of Table II give the calculated and observed shifts, in angstrom units, with respect to the smoothed wave-lengths of the second column. The $O-C$ values appear in the fifth column and are consistently positive. After excluding the last two lines, which are uncertain because of the perturbation, they average $+0.0057$ A $(= -0.038 \text{ cm}^{-1})$. The discrepancy may be due to an electronic shift of this magnitude and sign. The evidence is, however, not yet conclusive.

The wave-lengths of these isotope lines as they occur in the furnace spectrum were determined by one of us in 1921. It is interesting to note, however, that five of the six doublets (not two as stated in ref. 1, p. 32) had been observed, as single lines, by Kayser and

[1] *Astrophysical Journal*, **42**, 434, 1915.

[2] *Annalen der Physik*, **71**, 72, 1923.

Runge,[1] on arc spectrograms. The isotope $P(60)$ doublet is also definitely present on an extremely heavy arc spectrogram taken by one of us in the third order of the 21-foot grating of the University of Wisconsin. The $P(59)$ and $P(58)$ lines also are probably present, but are too hazy for measurement. The appearance of these CN isotope lines in the arc is important, since, aside from them, no lines involving the isotopes of oxygen, carbon, and nitrogen have as yet been found in the arc. This is discussed later in the paper.

Lines due to $C^{13}N^{14}$ have also been found in the (1,0) band. The head of the main band is at λ 3590.4, and of the isotope band at λ 3595.7. As usual, there is a decided difference between the arc and furnace spectrograms. In the arc a band structure decreasing in intensity toward the red, part of which is probably merged with the λ 3590 head, runs from this head to about λ 3675, beyond which it is obscured by the strengthening series of the λ 3883 band. This structure is due to the "tail" bands of cyanogen.[2] In the furnace it is almost completely suppressed, and in its place appears a faint band at λ 3595.7. The cyanogen spectrum, which in the vacuum furnace arises from the residue of air in the chamber, was strengthened by passing a stream of nitrogen through the furnace tube, with the chamber open to the air. This resulted in partly reversing the cyanogen bands and brought out the structure of the λ 3596 band in emission with satisfactory intensity. This structure is shown in the photometric curve c of Plate III, and also in Plate IVa. This latter figure is made up of three exposures. The middle one is the furnace at 2500° C, with nitrogen passing through at atmospheric pressure. Note that the head of the main band, at λ 3590, is self-reversed. This is precisely the state of affairs on the spectrogram in which we found the isotope lines in the (0,0) band. The upper and lower exposures represent the vacuum furnace, at 2800° and 2750° C, respectively. The upper shows the structure of the primary band, while in the lower, more strongly exposed, the isotope is

[1] *Abhandlungen der Preussischen Akademie der Wissenschaften*, 1889. Kayser and Runge's wave-lengths are in the old Rowland system and average 0.16 A greater than Uhler and Patterson's values for the main band lines. Their five isotope lines ($K'' =$ 56, 57, 59, 60, and 61) are, respectively, 0.19, 0.215, 0.17, 0.205, and 0.155 A greater than our mean doublet values.

[2] See W. Jevons, *Proceedings of the Royal Society*, A, **112**, 407, 1926, and F. A. Jenkins, *loc. cit.*

PLATE IV

3590.4   3595.7

4726   4737   4744.5
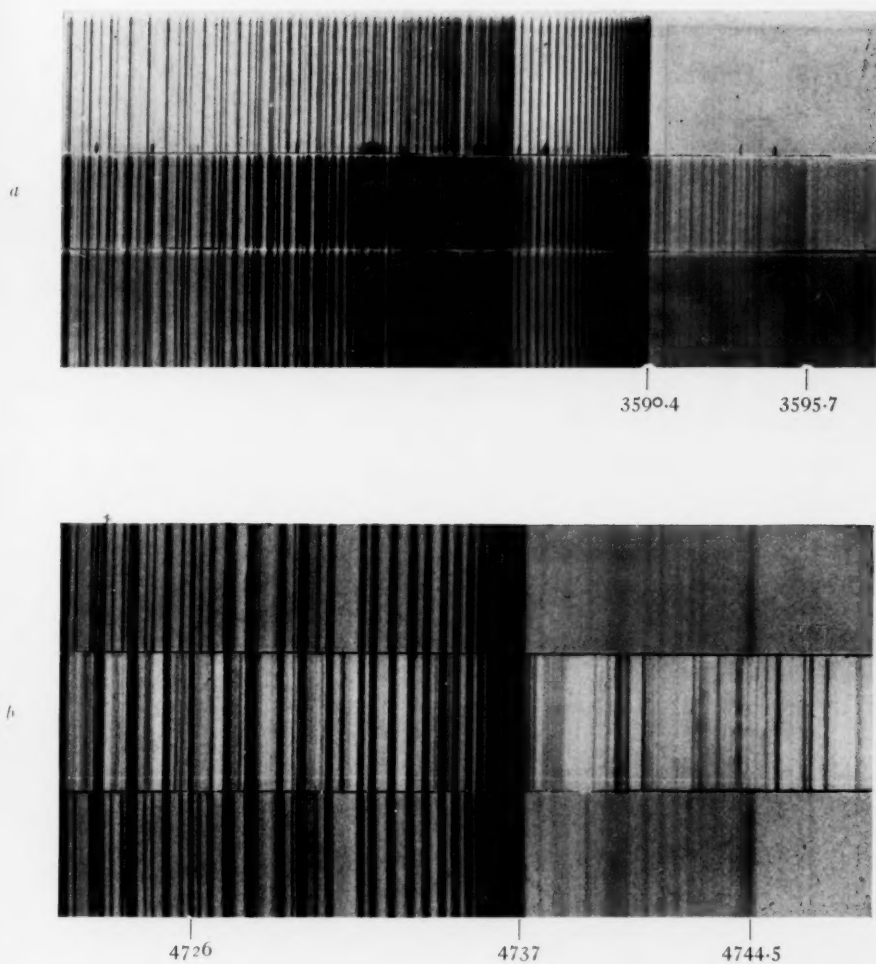
BAND STRUCTURES WITH ISOTOPES

a) Cyanogen band λ 3590 given by nitrogen in furnace (center), compared with vacuum furnace.

b) Furnace spectra of λ 4737 and isotope (above and below), compared with arc spectrum (center).
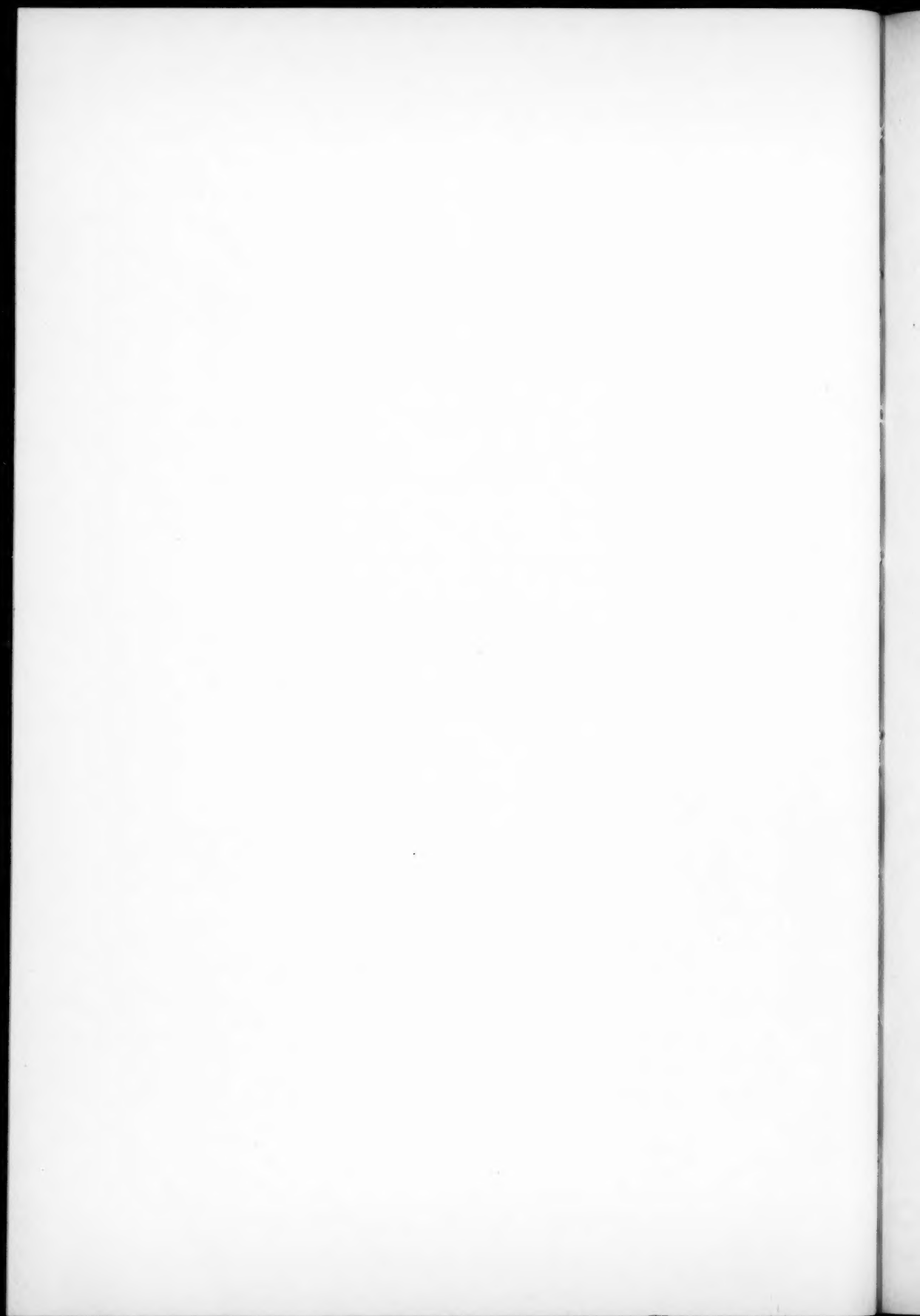
obviously present, but much blended with other lines, a condition presumably due to the CN tail bands. The calculated shift at the origin (27,921.44 cm^{-1}) is -44.54 cm^{-1} ($=5.69$ A). At the head it is about -41.1 cm$^{-1} = +5.30$ A. It has been possible to identify a number of lines in the isotope band, and for these the measured shifts average about 0.025 A (0.19 cm^{-1}) greater than the calculated values. There are, however, no good high-dispersion measurements for this band, and hence no really trustworthy constants.

In view of the more recent discovery[1] of an isotope of nitrogen of mass 15 in the absorption spectrum of nitric oxide, it is important to point out that the observed lines in CN cannot be due to a $C^{12}N^{15}$ molecule. For such a molecule the shift would be only 74 per cent of that due to $C^{13}N^{14}$, and the discrepancies in the (0,0) band would therefore run from 0.04 to 0.07 A, as compared with those given in the fifth column of Table II. In the (1,0) band the discrepancies would average 1.4 A. In concluding this discussion of the CN isotope bands, it may be pointed out that all of the calculations naturally assume that the main bands are due to the diatomic molecule $C^{12}N^{14}$, and the results verify this assumption. In this particular case, the identity of the carrier had already been established in other ways, but it should not be forgotten that, in general, quantitative proof in regard to this highly controversial matter (identity· of carriers of band spectra) is furnished only by means of isotope relations, as was emphasized by Mulliken[2] a number of years ago.

INTENSITY OF ISOTOPE BANDS, AND THE RELATIVE ABUNDANCE OF C^{12} AND C^{13}

The high intensity of the isotope band λ 4744 in the furnace as compared with the arc is a matter of special interest. Spectrograms previously taken of the Swan bands in the arc were examined and other arc exposures taken to decide if the band shows at all. Electrodes for the arc spectra were either commercial carbons or rods of Acheson graphite, the material of the furnace tubes. The appearance of the carbon bands in general is quite different in furnace and arc, as some of the fainter arc series are much strengthened in the furnace. Since in the present case it was necessary to have the λ 4737 band

[1] S. M. Naudé, *Physical Review*, **34**, 1498 (December 1), 1929.

[2] R. S. Mulliken, *ibid.*, **25**, 119, 1925.

in the arc fully as strong as in a furnace showing the isotope, the distinctive P series was selected as best representing the strength of the (1,0) band. Experiment showed that the furnace vapor at high temperature is very rich in the emitter of the Swan bands, since a relatively short furnace exposure gives the bands with high intensity and with some of the weaker arc series much strengthened. However, an exposure of the carbon arc ten times as long as that of the furnace at 2900° C gave the main series of the λ 4737 band fully as strong as in the furnace spectrum on the same plate. In fact, the arc spectrum came up first in the developer, indicating greater photographic density of the series of which the λ 4737 head is chiefly composed. The region where the isotope band should fall has some scattered band lines in the arc, apparently belonging to the strong (0,0) band at λ 5165, but the location of the isotope head λ 4744.5 falls in a clear place and the head was distinctly absent from the arc spectrum, as were such of the component isotope lines as should have been undisturbed by blends. Plate IVb shows two second-order furnace spectra of λ 4737 and its isotopes, and between them a spectrum of the band in the arc. The lower furnace spectrum has the lines of its main series comparable in strength with those in the arc band, but the isotope is missing from the arc spectrum. The stronger furnace spectrum at the top shows, by its large scale, that the lines of the isotope band are in general complex, as are those of the primary band.

Arcs in a partial vacuum, at pressures from 1 to 10 cm, also failed to show the head at λ 4744. The experiments with the arc, while not carried far enough to say that this source is quite unable to produce the λ 4744 band, show that the arc radiation is at a decided disadvantage in exciting the $C^{13}C^{12}$ molecule, whereas the furnace, with a long column of vapor giving the Swan spectrum with high intensity, including many of its fainter series, produces the isotope band with comparative ease. That the high temperature of the arc is unfavorable for the isotope radiation is probable, but as yet unproved. The range of furnace temperatures in which the Swan spectrum comes out is not large enough to show whether low temperature favors the isotope at the expense of the main band. At the high furnace temperatures much more carbon vapor is present and all of the phenomena are more pronounced.

Attempts were made to obtain the band at λ 4744 in absorption to test the possibility that a difference in absorption coefficient might explain the relative strength of the isotope band in the tube furnace. On account of the high furnace temperature required to reverse the Swan bands, the λ 4737 head was obtained only faintly in absorption when photographed with a diaphragm in the tube, and the λ 4744 head did not appear. It should have appeared, however, if its strength in absorption were one-fiftieth that of the primary head, or perhaps still fainter.

It has already been noted that certain $C^{13}N^{14}$ lines do appear in the carbon arc, but apparently with much less intensity, compared to the main bands, than in the furnace. It was thought at first that the failure to obtain the isotope bands in the arc was due to the fact that they were lost in the general intense background which develops on an overexposed arc spectrogram. This factor limits the allowed intensity of the main band, but evidently does not explain the great weakness of the isotope lines, as is proved by Plate IVb. The λ 4737 Swan band has never been obtained in the arc with the intensity with which the λ 3883 CN band can easily be photographed. Hence the failure of the isotope band at λ 4744 to appear in the arc is not inconsistent with the occurrence of $C^{13}N^{14}$ lines in the arc band λ 3883. Rather, the conclusion would be that while C^{13} is present in the arc vapor, its excitation in this source is difficult.

Before considering the possible reasons for this condition of affairs, we shall report a first crude attempt to measure the relative intensity in the furnace of the $C^{12}C^{12}$ and $C^{13}C^{12}$ bands. This was done by using an exposure ratio which gave the λ 4737 and λ 4744 bands with approximately the same intensity, according to a visual estimate. The actual exposure ratio was 100 to 1. We estimate, however, that half of the intensity of λ 4744 is due to the background, giving 200 to 1 as the final estimate. Now if one has two kinds of atoms A and B, with a relative abundance (A to B) of r, and capable of forming with equal facility the molecules AA, AB, and BB, it may easily be shown from elementary statistics that the relative abundance of the three types of molecules will be $r^2 : 2r : 1$. Hence the ratio of AA to AB is $r/2$, which in our case is 200 to 1. Therefore $r = 400$. If, as we have indicated earlier in the paper, the atomic weights of A and B are probably 12 and 13, even, the atomic weight

of the mixture should be 12.0025. This is the best chemical value at the present time.[1] The apparent abundance of C^{13} in the furnace is thus at least not inconsistent with the chemical atomic weight of carbon.

The whole question of relative abundance is, however, rendered uncertain if the strength of the isotope relative to the primary band depends on the excitation conditions. Thus we have found it many times less abundant in the arc than in the furnace. On the other hand, Sanford[2] has pointed out that the λ 4744 band occurs in N-type stars with a relatively high intensity. Menzel[3] has later estimated this relative intensity of λ 4737 to λ 4744 as not over 5 to 1, giving r as not more than 10. In this case a band due to $C^{13}C^{13}$ may be expected to appear, and in fact does so, as discussed by Sanford and Menzel. It has recently been noted by Bobrovnikoff[4] that the λ 4744 $C^{13}C^{12}$ band occurs also in certain comets.[5] The λ 4744 "line" found by Raffety, to which reference has been made,[6] appeared in the Mecker flame spectrum. In Johnson's reproduction,[7] in which it also appears, the source is a vacuum tube. In both these cases the distribution of intensity among the series lines proves that the effective temperature is low, probably only a few hundred degrees Centigrade in the case of the tube. The relative intensity of the λ 4744 and λ 4737 bands in the Mecker flame appears from the reproduction to be about that in our furnace spectrograms. In Johnson's plate λ 4744 seems stronger.

Why the relative intensities of the Swan band λ 4737 and its isotope λ 4744 should vary greatly in different sources is as yet by no means clear, but we may consider the condition as it stands and some of the factors which may influence it.

The relative intensity of the isotope band increases enormously, as just noted, in passing from arc (emission), to furnace (emission) or Mecker flame or vacuum tube (both emission), to star (absorption). The more obvious differences in these sources which might

[1] Birge, *Physical Review*, Suppl. 1, 1, 1929; see p. 26.

[2] *Publications of the Astronomical Society of the Pacific*, **41**, 271, 1929.

[3] *Ibid.*, **42**, 34, 1930. [4] *Ibid.*, p. 117, 1930.

[5] See, for instance, W. H. Wright, *Lick Observatory Bulletin*, No. 209: Comet Brooks (1911c).

[6] Raffety, loc. cit. [7] *Loc. cit.*

affect this ratio are (1) differences in relative abundance of the atoms C^{12} and C^{13}; (2) excitation differences, probably chiefly in temperature; and (3) the large differences in optical path, which might result in favoring the less abundant isotope.

In the laboratory experiments, the furnace tube and the arc electrodes have been of the same material, Acheson graphite, so that a difference in abundance must result from an easier dissociation in the arc of the $C^{13}C^{12}$ molecule, rather than from the number of each sort of molecule initially present.[1] In the star, a much larger proportion of one kind of atom would be contrary to the hypothesis of the uniformity of matter in the universe—a view which has gained strength since the recent ionization theory has explained many apparent contradictions in stellar spectroscopy.

Considering the possible influence of excitation, we find that the three sources—arc, furnace, and star—decrease in temperature in the same order that the strength of λ 4744 increases. The Mecker flame and the vacuum-tube spectra seem to fit into this same temperature series. This would indicate that lower temperature favors the radiation of the molecule containing C^{13}. The mechanism of this, however, is not obvious. Since the difference between the two atoms rests on the nuclear structure of each, we cannot draw a close analogy to the familiar effects of temperature changes on line spectra, based on electronic transitions. If higher temperature dissociates the heavier molecule more readily, this would result in the change of abundance mentioned in the preceding paragraph; but no definite reason to expect such a dissociation has appeared. While the favorable furnace temperature for the appearance of λ 4744 is higher than the estimated temperature of the absorbing atmosphere of an N-type star, the difference of three or four hundred degrees would not seem large enough to give the greatly increased strength of the isotope band in the star. The temperature range of the furnace does not permit a direct test of the question. The Swan band λ 4737 appears faintly at about 2400° C. At 2600° C enough carbon is

[1] It should be pointed out in this connection that J. L. Hundley, *Physical Review*, **30**, 864, 1927, in the case of lithium, found a different "work function" for the two different atomic species, such that the relative abundance of Li^7/Li^6 decreased with increasing temperature. It does not seem possible, however, that such an effect could account for the magnitude of the differences we have found.

vaporized to allow the isotope band to be photographed, and from this stage to 3000° C both bands strengthen rapidly. No distinct change in relative intensity appears within this range, and above 3000° C the strong continuous ground makes the comparison still more uncertain.

Considering the third possibility, we find the thickness of the optical path in arc, furnace, and stellar atmosphere increasing in the same order as the strength of the isotope band λ 4744 in the three sources. The arc vapor is a few millimeters thick, the heated portion of the furnace tube is 20 cm long, and the reversing layer of the star is of unknown but great thickness. To connect the increasing strength of the isotope band with increase of optical path would mean that the radiation in which C^{12} only is concerned reaches a maximum stage, beyond which increase of the column of vapor is ineffective, while for the less-abundant C^{13}, the longer column of vapor produces an effect more nearly in proportion to the number of molecules present. As in the case of the possible effect of temperature difference, we can only consider difference of optical path as one of the features which may influence the relative strength of these bands.

To summarize briefly, this paper shows that faint band structures appearing in various different regions of the spectrum are accounted for by the presence of a carbon atom of mass 13. The questions which have arisen as to abundance of this isotope and the dependence of its spectrum upon excitation conditions will require much additional evidence.

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June 1930

THE SPECTROGRAPHIC ORBIT OF
RS CANUM VENATICORUM[1]

By ALFRED H. JOY

ABSTRACT

The orbit of RS Canum Venaticorum is based on 35 spectrograms obtained with the 60- and 100-inch reflectors. Schneller's elements (Min. $=$ J.D. 2423579.344 H.G.M.T.$+$ $4^d797944E$) were used in the computation of phases. The lines of both components are visible in the spectrograms. Table I gives the observations, velocities, weights, and residuals. The spectrum of the primary star is F4, that of the secondary is dG8. The secondary is best observed at total eclipse, which lasts 3.7 hours. The spectroscopic absolute magnitudes are 3.5 and 4.5, respectively, and the parallax is $0''.010$. The elements of the orbit are: $e = 0.0$ (assumed); $K_1 = 91.6$ km/sec.; $K_2 = 99.0$ km/sec.; $\gamma = -8.9$ km/sec.; $a_1 \sin i = 6,040,000$ km; $a_2 \sin i = 6,530,000$ km; $m_1 \sin^3 i = 1.79$; $m_2 \sin^3 i = 1.66$.

The absolute dimensions were determined with the aid of B. W. Sitterly's photometric solution for the inclination and radii of the stars. The fainter star is exceptionally large and massive for its absolute magnitude and spectral type.

The eclipsing variable star RS Canum Venaticorum ($13^h5^m59^s$, $+36°28'$, 1900) was discovered in 1914 by Mme L. Ceraski. The spectral type given in the *Draper Catalogue* is F8. Spectrographic observations were begun at Mount Wilson in 1921 to determine the orbits and the spectrum of the fainter companion. Sitterly's photometric orbit,[2] based on M. Maggini's[3] observations and period, showed that the eclipses are total or annular, and that the surface brightness of the fainter companion is considerably less than that of the principal star. In the following year Sitterly stated in a letter that his observations at Princeton indicated that the double period found by C. Hoffmeister[4] should be used rather than that of 2.338433 days given by Maggini. He suggested the elements Min. $= 2422811.666$ G.M.T. $+4^d797851E$, which were the basis of a preliminary solution.[5] Dr. B. W. Sitterly has kindly permitted me to use the complete report of his observations and his computation of the photometric orbit, which will soon appear as a *Contribution from the Princeton University Observatory.*

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 403.

[2] *Astrophysical Journal*, **53**, 99, 1921.

[3] *Pubblicazioni del R. Osservatorio di Arcetri*, **34**, 64, 1916.

[4] *Astronomische Nachrichten*, **208**, 258, 1919.

[5] *Publications of the Astronomical Society of the Pacific*, **34**, 221, 1922.

In 1927 and 1928 H. Schneller[1] made at Neubabelsberg a series of 139 photographic observations from which he deduced photometric elements.

Since the preliminary solution was published in 1921, further spectrographic observations have been made at the 60- and 100-inch telescopes to strengthen the determination of the orbits and to add to the evidence concerning the spectrum of the fainter star, which is well seen for 3.7 hours during the time of total eclipse.

Thirty-five spectrograms have been measured for velocity as listed in Table I. These were obtained with a single prism and an 18-inch camera except C1112, 1687, 2301, 2935, 3339, 3664, 3798, 3868, 4663, 5173, and γ17220 and 17480, which were taken with a 10-inch camera.

Phases were computed from Schneller's elements,

$$\text{Min.} = \text{J.D. } 242\overset{.}{3}579 \cdot 344 \text{ H.G.M.T.} + 4\overset{d}{.}797944 \, E,$$

which are based on all the minima thus far observed. The weights assigned to each plate depend upon the dispersion, the number of
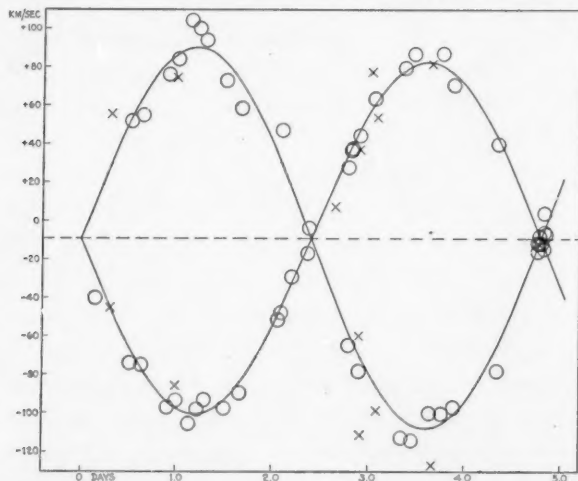
Fig. 1.—Velocity-curve of RS Canum Venaticorum. Crosses are preliminary measures of Victoria spectrograms.

lines measured, and the quality of the spectrum. The observations are plotted in Figure 1.

[1] *Astronomische Nachrichten*, **233**, 365, 1928.

Dr. J. S. Plaskett made eight spectrographic observations of the star at Victoria in 1921 and 1922. His rough preliminary measures of velocity are represented by crosses in the figure. One weak plate

TABLE I

OBSERVATIONS OF RS CANUM VENATICORUM

PLATE	DATE J.D. 242+	PHASE	PRIMARY			SECONDARY	
			Vel.	Wt.	O−C	Vel.	Wt.
			km/sec.		km/sec.	km/sec.	
C 1758	3238.705	0.015	....	....	....	− 5.9	0.5
4663	5277.833	0.017	....	....	....	− 14.1	.5
5181	5752.833	0.020	....	....	....	+ 4.3	.2
5173	5728.851	0.028	....	....	....	− 6.5	.5
γ 17220	6016.840	0.141	− 39.9	....	....	....	....
10093	2797.785	0.506	73.8	1.0	− 7.9	+ 52.2	.5
10095	2797.903	0.624	74.4	0.5	+ 1.9	+ 55.4	.2
11796	3565.842	0.892	96.9	0.5	− 3.8	+ 76.4	0.5
C 1761	3239.678	0.988	93.1	1.0	+ 3.8	+ 84.2	1.0
3664	4540.055	1.122	105.2	0.5	− 5.3	+104.5	0.2
3864	4693.674	1.206	98.0	.5	+ 2.5	+100.4	.5
3865	4693.747	1.280	92.8	.5	+ 7.3	+ 94.4	.2
2935	4012.651	1.492	97.6	.5	− 3.6	+ 73.2	.5
γ 13413	4276.698	1.652	89.6	.5	− 9.3	+ 58.5	.5
C 1764	3240.736	2.046	51.3	0.5	− 1.9	....	....
3364	4334.700	2.079	47.3	1.0	− 0.1	+ 47.5	.5
1803	3269.661	2.184	28.8	1.0	+ 5.2	....	....
3868	4694.816	2.349	16.5	0.5	+ 0.1	....	....
3339	4315.792	2.363	− 3.5	0.5	+11.0	....	....
3286	4277.819	2.774	+ 28.0	1.0	− 5.7	− 64.3	.5
4284	5040.719	2.800	36.6	1.0	+ 1.1	....	....
1635	3135.944	2.809	37.7	0.2	− 0.3	....	....
γ 13432	4306.722	2.889	44.9	1.0	− 0.4	− 78.0	.5
C 2007	3448.042	3.040	64.0	0.2	+ 4.7	....	....
γ 11667	3505.928	3.351	79.9	0.2	+ 2.1	−112.6	.1
C 2197	3534.815	3.450	86.9	1.0	+ 6.1	−114.3	0.5
γ 10063	2776.907	3.618	79.1	1.0	− 3.5	− 99.9	1.0
C 1639	3136.880	3.745	87.2	1.0	+ 6.3	−100.6	0.5
1112	2882.711	3.867	71.0	0.5	− 6.0	− 97.0	.2
γ 10085	2796.806	4.324	+ 40.1	1.0	− 4.2	− 78.0	.5
C 3798	4658.839	4.755	....	....	....	− 15.8	.2
1687	3185.870	4.756	....	....	....	− 11.1	.2
1544	3089.922	4.766	....	....	....	− 8.0	.5
2301	3593.710	4.771	....	....	....	− 11.7	.5
γ 17480	6131.844	4.791	....	....	....	− 11.3	0.5

has been rejected. It will be seen that the Victoria observations confirm the Mount Wilson results for the most part, but for the sake of homogeneity they were not included in the solution.

Because of rotation of the principal star, the velocity from plate γ17220, which was taken during partial eclipse, falls 15 km/sec.

below the curve and has been omitted from the solution. The expected rotation effect at the limb is 17 km/sec.

The spectral type of the more massive and brighter star is F4n. The lines are easily measurable but are not of the best quality.

When the differences of velocity are sufficient, the spectra of both stars appear, but the lines of the secondary are weak and unsatisfactory for accurate measurement. During totality, however, the spectrum of the secondary is easily observed. Nine spectrograms which show the spectrum of the secondary entirely free from interference by the light of the primary star were obtained at this phase. The spectral type is G8, and the usual criteria indicate definitely that it belongs to the dwarf branch.

The spectrographic absolute magnitudes determined with the cooperation of Dr. W. S. Adams are 3.5 for the primary and 4.5 for the secondary. The corresponding apparent visual magnitudes found by Sitterly by comparison with the HP star B.D.$+36°2352$ are 8.56 and 9.42, respectively. The parallax from these values is $0.''010$.

A velocity-curve drawn through the observations indicates that within the errors of the observations the orbits are circular. Hence, the eccentricity was assumed to be zero. Corrections to the estimated values of γ and K for the primary orbit were determined by least squares. The probable errors given result from this solution. It was not considered worth while to include the observations of the secondary; γ was not derived for the secondary orbit.

The elements thus found are

$$P = 4^{d}797944 \text{ (Schneller)} \qquad a_1 \sin i = 6{,}040{,}000 \text{ km}$$
$$e = 0.0 \text{ (assumed)} \qquad a_2 \sin i = 6{,}530{,}000$$
$$K_1 = 91.6 \pm 0.10 \text{ km/sec.} \qquad m_1 \sin^3 i = 1.79 \odot$$
$$K_2 = 99.0 \qquad m_2 \sin^3 i = 1.66$$
$$\gamma = -8.9 \pm 0.21 \qquad m_2/m_1 = 0.93$$

The computed curve is shown in Figure 1, and the residuals appear in the sixth column of Table I. Except for some uncertainty in the exact value of i resulting from the various photometric solutions, the masses seem fairly well determined. Both stars are dwarfs with nearly equal masses, approximately 1.7 times that of the sun. The stars do not agree well with Eddington's mass-luminosity curve, which requires an absolute magnitude of $+2.1$ for such a mass.

PLATE V

SPECTROGRAMS OF RS CANUM VENATICORUM

a) C1761, double lines, primary displaced to violet
b) C1639, double lines, primary displaced to red
c) C1803, primary star, spectrum F4
d) C1544, secondary star taken during total eclipse, spectrum G8
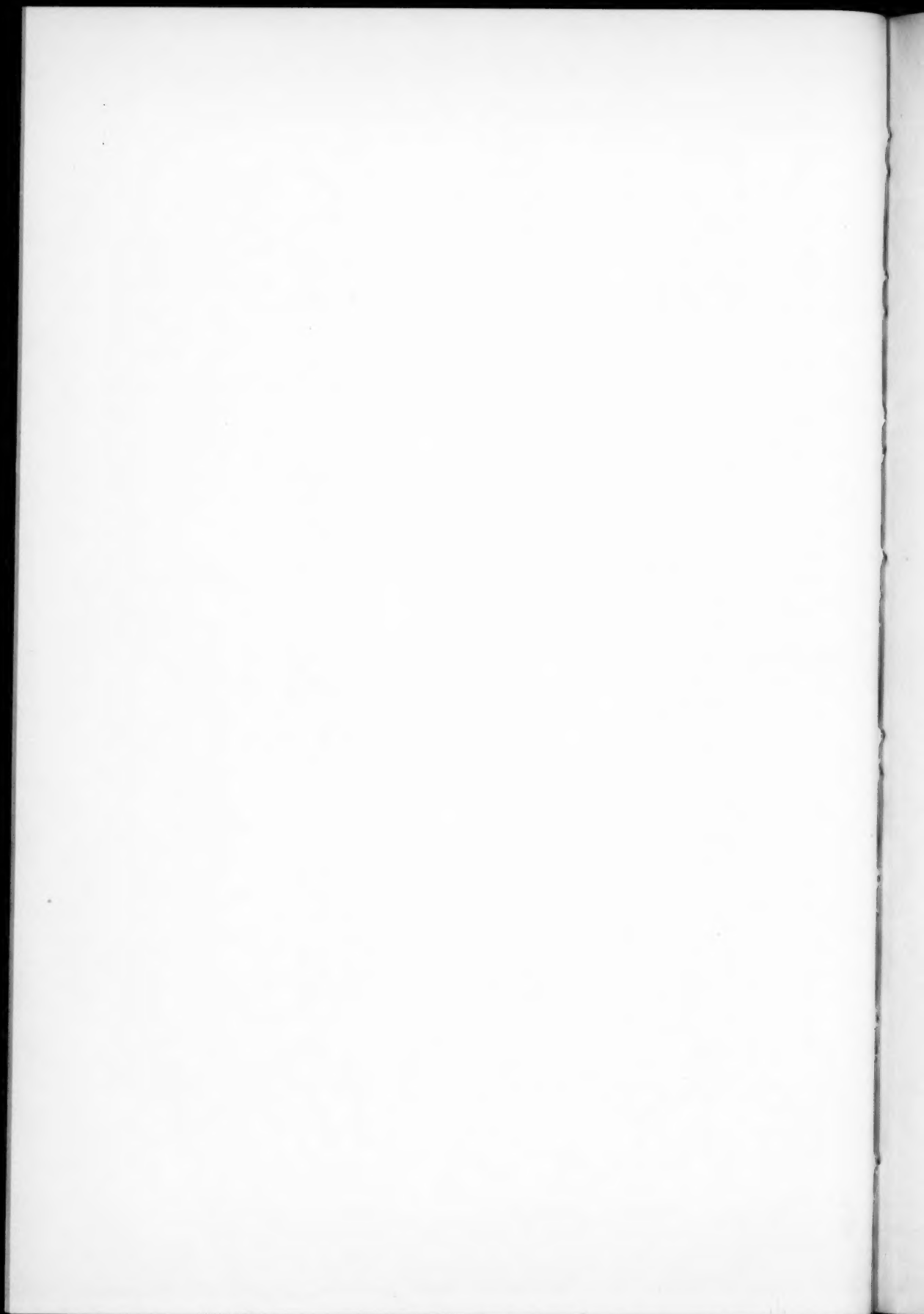
Using $i = 79°9$, $r_f = 0.289$, $r_b = 0.087$, from Sitterly's solution II (uniform disk), we have for the absolute dimensions

$a_1 + a_2$, semi-major axis of relative orbit........12,780,000 km
a_1, semi-major axis of primary orbit...........6,140,000
a_2, semi-major axis of secondary orbit.........6,640,000
r_b, radius of brighter star....................1,110,000 (1.6⊙)
r_f, radius of fainter star.....................3,680,000 (5.3)
m_b, mass of brighter star......................   1.85⊙
m_f, mass of fainter star.......................   1.71
ρ_b, density of brighter star....................   0.45⊙
ρ_f, density of fainter star.....................   0.012

The secondary star is certainly peculiar and cannot readily be classified among other stars for which we know the physical characteristics. Its absolute magnitude and spectrum would place it among the dwarfs, but the size and mass are exceptionally large, although not nearly large enough to compare with the normal giants of its spectral class. The density is, correspondingly, intermediate between dwarfs and giants. Its large size is checked by the fact of the long total eclipse during which a distinctly later-type spectrum shows.

VARIATIONS IN RADIAL VELOCITY OF THE CEPHEID VARIABLES W GEMINORUM, U AQUILAE, AND DT CYGNI[1]

By ROSCOE F. SANFORD

ABSTRACT

Variation in radial velocity.—Curves of radial velocity for W Geminorum, U Aquilae, and DT Cygni have been derived from Mount Wilson spectrograms and a series for U Aquilae obtained at the Lick Observatory (top curves of Figs. 1, 2, and 3). The semi-amplitudes of variation in velocity are 16.7, 20.8, and 7.5 km/sec., respectively. The velocities of the systems are −0.7, −7.0, and −0.5 km/sec.

Relation of velocity-curves to light-curves.—The velocity-curves of U Aquilae and DT Cygni are mirror images of the light-curves, i.e., maximum velocity coincides with minimum light and minimum velocity with maximum light. For W Geminorum the interval between maximum and minimum velocity is nearly a day less than the mean interval between minimum and maximum light. Since the period derived from the radial velocities shows that maximum velocity coincides in the mean with minimum light, it follows that with this period minimum velocity precedes maximum light by nearly a day.

Superposed minor fluctuations are present in the light-curves of W Geminorum and U Aquilae. Evidence for kindred fluctuations in the velocity-curves is slight for the former and lacking for the latter.

This paper deals with the variations in radial velocity of the three Cepheid variables listed in Table I. With the exception of some

TABLE I

Star	H.D. No.	B.D. No.	α 1900	δ 1900	Harv. Mag.	Sp.
W Geminorum...	46595	$+15°1246$	6^h29^m2	$+15°25'$	6.7–7.5	cG0
U Aquilae.......	183344	− 7 4698	19 24.0	− 7 15	6.2–6.9	cF8
DT Cygni.......	201078	$+30 4318$	21 2.3	$+30 47$	5.9	F7

early values for U Aquilae, referred to later, all the radial velocities used depend upon spectrograms obtained with one-prism spectrographs attached to the 60- and 100-inch reflecting telescopes at Mount Wilson. Their dispersion is about 37 A per millimeter at $H\gamma$. Plates taken with the 60-inch are designated by γ, and those taken with the 100-inch telescope by C. Most of the wave-lengths of the absorption lines measured are from a list based upon measures of a number of plates of the Cepheid variable T Monocerotis.[2]

The curves of radial velocity have been formally represented by orbital elements by comparing the plot of the velocities with standard curves based on data given by E. S. King.[1] For DT Cygni, which shows little evidence of eccentricity, a simple sine-curve sufficed. The details for each star follow.

W GEMINORUM

E. F. Sawyer's[2] observations in 1895 and 1896 led him to announce the variability of this star in a period of nearly eight days. The light-curve shows definitely that W Geminorum is a Cepheid, which oscillates through a range of 0.5 mag. with a median visual magnitude of approximately 7.0. It is therefore bright enough at all times to give spectrograms of good density, with either the 60- or the 100-inch reflector, in reasonably short exposure times. As the number of such Cepheids is small, it seemed worth while to obtain spectra at all phases of the light variation. Observations to determine the variation in radial velocity were begun in 1927. Four spectrograms had already been obtained in 1917–1918, two near maximum light and two near minimum, which showed[3] that the velocities near maximum were approximately 32 km/sec. less than those at minimum, and that the spectrum is that characteristic of Cepheid variation. The Mount Wilson spectral classification is cGo.

Nineteen spectrograms of usable quality were obtained between January, 1927, and March, 1930. The data for all twenty-three spectrograms appear in Table II. The interval of more than ten years between the first four and the remaining observations and the distribution of the latter over three consecutive years should make it possible to derive the period (if constant) with considerable precision from the radial velocities alone. Accordingly, it was decided to use the radial velocities independently of the light observations.

The velocity-curve was judged to be smoothest when the twenty-three velocities were assembled with a period of $7^{d}91478$. A freehand curve through the plot, compared with typical velocity-curves, gave

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[1] *Harvard Annals*, **81**, 231, 1923.

[2] *Astronomical Journal*, **17**, 3, 1896.

[3] Adams, Joy, and Sanford, *Publications of the Astronomical Society of the Pacific*, **36**, 137, 1924.

elements, which, together with the period, were then corrected by
the least-squares procedure outlined by Schlesinger.[1]

TABLE II

W Geminorum—Observations of Radial Velocity

Plate No.	Date	G.M.T.	Phase	Vel.	O−C	Wt.
				km/sec.	km/sec.	
γ 6385......	1917 Nov. 25	23^h55^m	7^d152	+23.0	+5.4	1.0
6565......	1918 Jan. 4	22 31	7.517	+21.4	+3.1	0.5
6595......	Jan. 23	19 11	2.632	− 8.9	+0.1	1.0
6673.....	Jan. 31	18 01	2.668	− 9.9	−1.1	1.0
14793......	1927 Jan. 18	19 42	7.666	+17.6	+1.4	0.5
15368......	Nov. 2	22 43	2.920	−10.0	−2.3	1.0
15374......	Nov. 4	0 10	3.980	− 1.2	+1.8	1.0
C 4527......	Nov. 6	22 55	6.928	+11.7	−4.1	1.0
γ 15430......	Dec. 6	21 36	5.212	+ 3.4	+0.2	1.0
C 4572......	Dec. 7	20 56	6.184	+ 4.1	−5.4	0.5
4606......	1928 Jan. 3	19 55	1.480	− 9.3	+4.4	0.5
γ 15610......	Mar. 6	15 26	0.969	−16.8	−2.1	0.5
C 4722......	Apr. 6	15 07	0.295	− 2.3	+3.9	0.5
γ 16194......	Sept. 28	0 30	0.546	−12.7	−0.5	1.0
16283......	Nov. 5	23 36	7.846	+16.7	+6.4	1.0
16315......	Nov. 22	21 52	1.028	−21.3	−6.6	1.0
16380......	Dec. 31	16 55	0.245	− 7.0	−2.7	1.0
16383......	Dec. 31	23 02	0.500	− 9.1	+2.4	1.0
C 5153......	1929 Mar. 29	15 21	1.110	− 6.1	+8.6	0.5
γ 16503......	Mar. 31	16 00	3.317	− 6.6	+0.2	1.0
16506......	Apr. 1	16 49	4.171	+ 1.4	+3.5	0.5
17320......	1930 Mar. 11	15 27	7.751	+ 7.1	−6.8	0.5
17351......	Mar. 19	15 31	7.837	+ 9.1	−1.5	1.0

TABLE III

Elements of W Geminorum

Preliminary		Corrections	Adopted
P = Period	= 7^d91478.....	$+0^d00065$	7^d91543
K = Semi-amplitude of velocity variation	= 17.9.........	−1.17	16.7 km/sec.
e = Eccentricity	= 0.55........	−0.024	0.53
ω = Angle of periastron	= 70°0.........	+2°31	72°3
T = Time of periastron passage	= J.D. 2425612.395	+0.064	5612.460
γ = Velocity of the system	= + 0.23........	−0.93	−0.70 km/sec.

The preliminary elements, corrections, and adopted elements are
given in Table III.

The phases and residuals given in Table II were computed with
the corrected elements. Som.˙ of the residuals are rather large for

[1] *Publications of the Allegheny Observatory*, **1**, 33, 1910.

spectrograms as well measurable as these; but attention has already been called[1] to the fact that other Cepheids show a considerable scattering in the radial velocities around maximum light. Most of the large residuals for W Geminorum are near maximum and minimum; further, the representation by orbital elements is only a convenient device which defines the variation in velocity, perhaps to a first approximation only.

The elements show that maximum velocity precedes T, the periastron time, by 0^d52, and that minimum velocity follows T by 1^d00.

It now becomes of interest to locate the maximum and minimum velocities with reference to light minimum and maximum. The star

<div align="center">TABLE IV</div>

Obs. Light Max. (L max) G.M.T.	L max $-$ L min $(M-m)$	T (Periastron Passage)	L max $-T$	L min $-T$	Authority	Interval
J.D. 2413266.35..	$+2^d91$	64.39	$+1^d96$	-0^d95	Luizet*	1895–1905
9029.00..	2.20	26.82	2.18	0.02	Van der Bilt†	1907–1914
2420073.25..	2.00	71.66	1.59	0.41	Dziewulski‡	1912–1915
0682.57..	$+2.53$	81.15	$+1.43$	-1.10	Robinson§	1895–1929
Mean.......	$+2.41$	....	$+1.79$	-0.62	....	....

* *Astronomische Nachrichten*, **169**, 401, 1905.
† *Journal des observateurs*, **9**, 23, 1926.
‡ *Bulletin de l'Observatoire de Vilno*, No. 4, 1924.
§ *Harvard Bulletins*, No. 872, 13, 1930.

has been observed photometrically by many, but only four have determined epochs of light minima and maxima entitled to considerable weight, either because their own observations are extensive or because their results are based upon several series of observations. The necessary data are in the first two columns of Table IV, which also shows, in the third column, the time of nearest periastron passage. The combination of these three columns gives the results in the fourth and fifth columns, which, in the mean, are L max $-T = +1^d79$, L min $-T = -0^d62$.

Since maximum velocity precedes T by 0^d52 and minimum follows T by 1^d00, the orbital elements represent maximum velocity as coinciding closely on the average with minimum light, while minimum velocity precedes maximum light by 0^d79.

[1] *Mt. Wilson Contr.*, No. 340; *Astrophysical Journal*, **66**, 177, 1927.

As they stand, the results for W Geminorum point to a definitely shorter interval between the extremes of radial velocity than between those of light, the one being 1^d52 in the mean and the other 2^d41 (see Table IV, col. 2). A somewhat shorter period, or one that undergoes a slight secular increase, would reduce the mean difference L max $-T$ from 1^d79 to 1^d00, or to the interval between T and velocity minimum; but the mean difference $T - L$ min would then no longer accord with $T -$ Velocity maximum.

The period required to reduce L max $- T$ to 1^d00 is 0^d00021 shorter than that adopted in Table III, and is nearly midway between that obtained from the radial velocities and the results of W. Dziewulski and of L. V. Robinson, 7^d91496 and $7^d914995$, respectively, which are based upon the longest intervals of photometric observations.

One way of accounting for the anomaly would be to assume that the representation of the radial velocities by orbital elements places velocity minimum too early, allowance for which would increase the interval between velocity maximum and velocity minimum. The evidence that the velocity-curves of other Cepheids are mirror images of the light-curves rests, however, upon velocity-curves defined by orbital elements. For a strict comparison, therefore, it is necessary to abide by the representation adopted. Whether the period should be that which makes light minimum and velocity maximum coincide, or the reverse, can be decided only by simultaneous observations of light and velocity. Since these are lacking, the period derived from the radial velocities has been retained.

The velocities and the velocity-curve based on the adopted elements are shown at the top of Figure 1. Each velocity is plotted as a circle with a radius of 2.5 km/sec.; those for 1917 are barred. The velocity of the center of mass is represented by the broken horizontal line at the ordinate -0.7 km/sec.

Below, in order from the top, are the light-curves of J. van der Bilt, M. Luizet, Dziewulski, and Robinson. The scales of ordinates at the left and right are in magnitudes and refer to the curves of van der Bilt and Robinson, respectively. The observations of Luizet and Dziewulski are plotted with approximately the same amplitude as the other curves, but bear no relation to either the left- or right-

hand ordinates. All have been arbitrarily placed so that minimum
light matches maximum velocity.
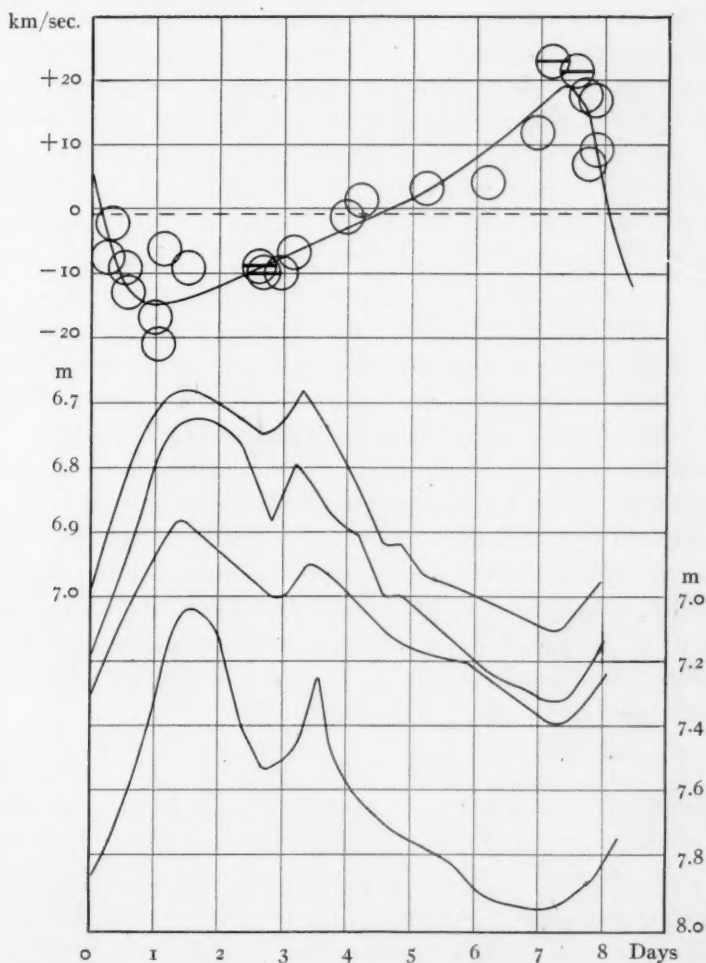
FIG. 1.—Top: Velocity-curve of W Geminorum. Circles represent individual
Mount Wilson observations with phases reckoned from periastron. Below this are the
light-curves of van der Bilt, Luizet, Dziewulski, and Robinson, respectively. Left-
and right-hand ordinates belong to top and bottom light-curves, respectively. Robin-
son's curve is based on photographic magnitudes, the others on visual estimates.

Dziewulski's and van der Bilt's curves include only their own ob-
servations; Luizet's is based upon the data of several observers
covering a period of ten years; and Robinson's is constructed from

photographic magnitudes observed during most of the interval since the star has been known to be a variable.

In spite of the diversity of the data, the curves strongly resemble each other in several respects. The values for $M-m$ in Table IV, which average $2^{d}.41$, show some spread, but none is as small as the interval between maximum and minimum velocity, $1^{d}.52$. A conspicuous common feature is the secondary minimum $1^{d}.4$ after light maximum, which is especially marked in Robinson's photographic light-curve. All four curves appear also to have a slight though definite pause or inflexion $3^{d}.4$ after light maximum. It should be remarked, however, that E. C. Pickering's[1] light-curve shows a smooth, continuous variation with little evidence of minor fluctuations; but this one determination, excellent though it may be, can scarcely throw doubt upon the reality of features common to the four well-determined curves illustrated in Figure 1. The absence of minor fluctuations from Pickering's curve may perhaps result from the manner of combining observations to form the mean curve.

Although the data for radial velocity are rather meager, it is perhaps worth while to call attention to apparent pauses in the upward trend of the velocities which are roughly in agreement with the two inflections in the light-curves. Many more observations, however, especially during the phase interval from $1^{d}.5$ to $2^{d}.5$, are needed to settle this point definitely.

<center>U AQUILAE</center>

This variable, discovered by Sawyer[2] in 1886, is a Cepheid with a period of about a week. Luizet[3] discussed the data accumulated between 1886 and 1905 and showed that the maxima could be represented by J.D. $2410170.325+7^{d}.02387E$ (G.M.T.). All the light-curves show a continuous variation with $M-m$ ranging from $1^{d}.98$ to $2^{d}.30$, the latter being Luizet's values. The range in visual magnitude for his curve is 6.2–6.9.

Sixteen spectrograms of U Aquilae were obtained at Mount Hamilton during 1902–1905. The star was one of the faintest of a number of Cepheids thus observed, and its spectrograms were not

[1] *Harvard Annals,* **46,** 155, 1903.

[2] *Astronomical Journal.* ,, 22, 1886.   [3] *Astronomische Nachrichten,* **171,** 265, 1906.

considered good enough to justify a detailed discussion of the velocity variation. Albrecht[1] noted, however, that the plates showed a range of 25 km/sec., and that the variation appeared to agree with the period of light variation. More will be said later of this series of radial-velocity determinations.

Two spectrograms obtained at Mount Wilson in 1918 showed a range of velocity of about 18 km/sec. A series of observations, be-

TABLE V

U Aquilae—Mount Wilson Observations of Radial Velocity

Plate No.	Date	G.M.T.	Phase	Vel.
				km/sec.
γ 7263.........	1918 Aug. 25	16^{h}00^m	1^{d}718	− 14.2
8704.........	Sept. 12	17 10	5.476	+ 8.8
16630.........	1929 June 14	20 33	0.491	− 21.2
16724.........	July 18	20 31	6.395	− 20.5
16733.........	July 20	20 20	1.363	− 11.8
16817.........	Aug. 18	16 36	2.071	− 5.0
16822.........	Aug. 19	15 58	3.086	− 4.9
16831.........	Aug. 20	18 06	4.175	+ 6.5
16839.........	Aug. 21	19 13	5.222	+17.3
C 5289.........	Sept. 10	16 20	4.030	+ 6.1
5294.........	Sept. 11	15 51	5.009	+14.1
γ 16902.........	Sept. 12	16 13	6.025	− 11.2
16907.........	Sept. 13	15 7	6.979	− 21.8
C 5346.........	Oct. 20	14 18	1.802	− 15.3
5444.........	1930 May 11	21 33	1.617	− 15.8
5449.........	May 12	23 30	2.698	− 6.9
5453.........	May 13	23 16	3.688	+ 5.6
γ 17492.........	June 5	20 57	5.520	+15.0
17495.........	June 6	21 57	6.562	− 20.7
17502.........	June 7	22 49	0.574	− 21.6
17510.........	June 8	22 20	1.554	− 14.9
17514.........	June 9	23 00	2.581	− 11.4

gun in 1929 and continued into 1930, includes twenty spectrograms fairly well distributed in phase. The twenty-two plates and the measured radial velocities are listed in Table V, with phases from maximum light, as computed with Luizet's formula.

The velocities, represented by circles with a radius of 1.5 km/sec., are assembled according to phase at the top of Figure 2. The two observations of 1918 are indicated by barred circles.

Through the kindness of Dr. J. H. Moore the spectrograms obtained at the Lick Observatory were loaned for this discus-

[1] *Publications of the Astronomical Society of the Pacific,* **18,** 142, 1906.

sion, where they have served a very useful purpose in determining
phases of the velocity-curve at an epoch more than twenty years
earlier than that of the majority of the Mount Wilson observations.
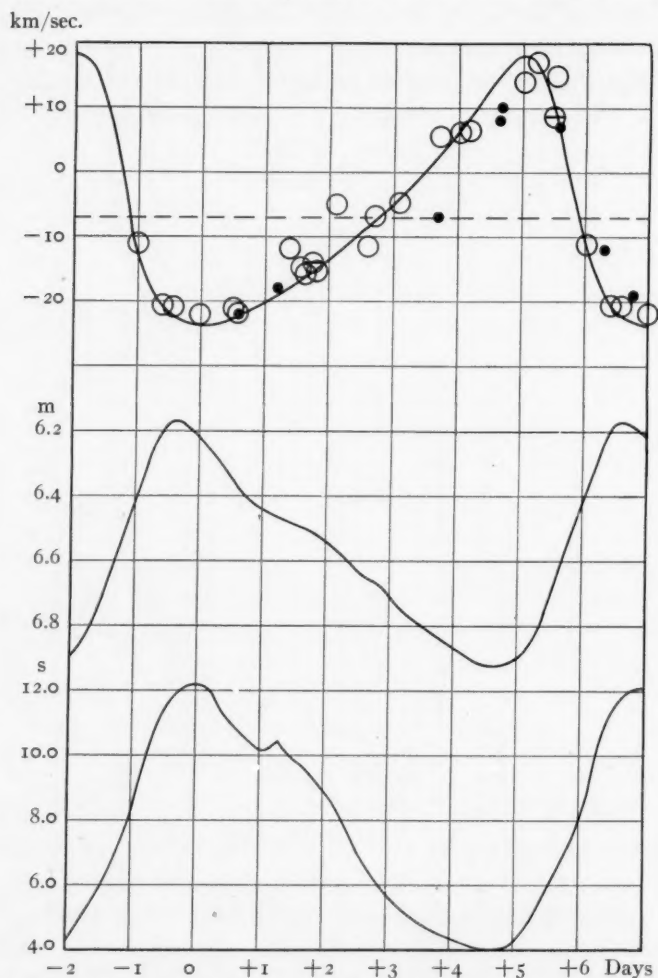
FIG. 2.—Top: Velocity-curve of U Aquilae. Circles represent individual Mount
Wilson observations; dots, Lick Observatory normal places. Phases are reckoned from
light-maximum. The broken horizontal line represents the velocity of the system.
Middle: Pirkering's light-curve. Bottom: Luizet's light-curve.

The data for normal places derived from fourteen of the Lick
Observatory spectrograms, measured and reduced by the writer, are
given in Table VI. The resulting velocities are plotted as solid circles

at the top of Figure 2. Pickering's[1] light-curve appears immediately below with the scale of ordinates in visual magnitudes on the left. Pickering's epoch of maximum light gives a residual of $-0^{d}37$ when compared with Luizet's formula. The curve has accordingly been shifted so that its maximum is at phase $-0^{d}37$. At the bottom of the figure is Luizet's light-curve with ordinates expressed in "steps."

TABLE VI

U Aquilae—Normal Places from Lick Observatory Radial Velocities

No.	Phase	Vel.	No. of Plates	Dates of Plates
		km/sec.		
I........	$0^{d}564$	-22	3	1904 July 12, 1905 May 17, 1905 Oct. 5
II.......	1.219	-18	2	1902 July 6, 1905 Oct. 20
III......	3.710	$- 7$	1	1905 Oct. 15
IV......	4.644	$+ 8$	2	1903 Aug. 14, 1905 Oct. 16
V.......	4.686	$+10$	2	1905 Oct. 9, 16
VI......	5.588	$+ 7$	2	1905 May 22, Oct. 17
VII.....	6.296	-12	1	1905 July 11
VIII....	6.769	-19	1	1905 Oct. 4

The use of Luizet's formula for deriving phases places maximum velocity in close agreement with minimum light and minimum velocity in coincidence with maximum light, and shows that Pickering's light-curve from observations in 1897–1898 agrees well with the data of 1886–1905 upon which Luizet based his curve. The radial velocities from the Lick Observatory spectrograms are also well placed by phases thus computed. Luizet's period must therefore be substantially correct.

TABLE VII

$$P = 7^{d}02378$$
$$K = 20.8 \text{ km/sec.}$$
$$e = 0.40$$
$$\omega = 60°$$
$$T = \text{J.D. } 2410168.860 \text{ (G.M.T.)} = \text{Max. light} - 1^{d}465$$
$$\gamma = -7.0 \text{ km/sec.}$$

Comparison of the distribution of the Mount Wilson radial velocities with standard velocity-curves for spectroscopic binaries gave the elements appearing in Table VII, which represent the observations so well that they have not been corrected by the method of least squares. The corresponding velocity-curve is shown in Figure 2. The broken line at -7 km/sec. indicates the velocity of the center of mass.

[1] *Op. cit.*, p. 141, 1903.

The interval between maximum and minimum velocity is almost precisely 2^d. This agrees closely with Pickering's value of $1^d.98$, but is slightly less than the mean of the results by Yendell, Pickering, Chandler, and Luizet,[1] namely, $2^d.19$.

The period of velocity variation for U Aquilae is therefore identical with the period of light variation, and light minimum coincides closely with velocity maximum and light maximum with velocity minimum. The steep part of the velocity-curve may, however, cover a slightly shorter phase interval than the corresponding part of the light-curve.

Both the light-curves illustrated in Figure 2 show some evidence of a pause in the slowly descending branch about a day after maximum. Since no radial velocities are available for the same phase after velocity minimum, the evidence for or against an inflection in the velocity-curve at this point is inconclusive.

The use of a period somewhat shorter than Luizet's would improve the agreement of the Lick and Mount Wilson observations by shifting the latter in Figure 2 to the right with respect to the former, but would still leave something to be desired. Agreement could also be obtained by retaining Luizet's period and assuming that the velocity amplitude in 1902–1905 was less than that at the time the observations at Mount Wilson were made. The 1902–1905 spectrograms were, however, difficult to measure, and it seems best to assume that they are in substantial agreement with the later observations and to conclude for the present that the character of the velocity variation has changed little, if any, during more than twenty years.

DT CYGNI

While observing T Vulpeculae with the photo-electric photometer, J. Stebbins and C. M. Huffer[2] found that one of the comparison stars was itself a variable. This star is known as DT Cygni. Huffer's discussion of their photometric observations obtained in 1925 and 1926 shows that phases computed by

$$\text{Max.} = \text{J.D. } 2424305.651 + 2^d.4993E \text{ (G.M.T.)} \qquad (1)$$

[1] Müller and Hartwig, *Geschichte und Literatur des Lichtwechsels*, **2**, 203, 1920.

[2] *Publications of the Washburn Observatory*, **15**, 132, 1928.

give a very satisfactory light-curve. The variation is continuous and symmetrical, and is well represented by a sine-curve with an amplitude of 0.385 mag.

After these results appeared, it was found that a mean velocity of −6 km/sec. had been published for the star from three spectrograms[1] taken at Mount Wilson in 1913, having individual values ranging between −2 and −9 km/sec.

Huffer's epoch and period placed all three observations within $0^d.35$ of light maximum. This raised the question whether the range
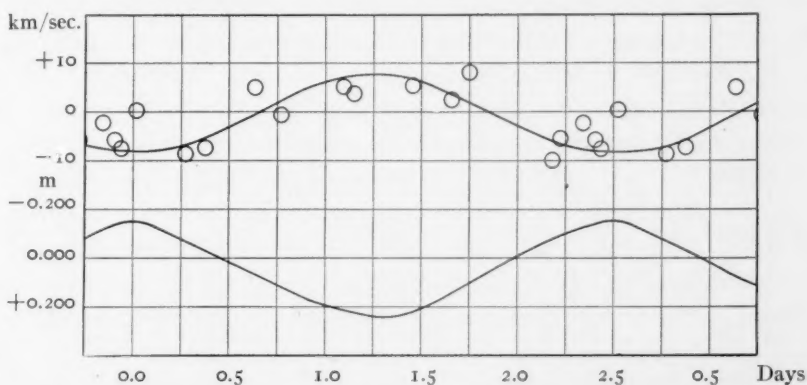
FIG. 3.—Top: Velocity-curve of DT Cygni. Circles represent individual Mount Wilson observations. Velocity of the system is −0.5 km/sec. Phases are reckoned from light-maximum. Bottom: Huffer's light-curve from observations made with the photoelectric cell. Ordinates are magnitude differences between DT Cygni and a comparison star.

in radial velocity might not be greater than that shown by the three plates. A test plate confirmed this suspicion; and, since the light variation is somewhat less than usual for Cepheids, it seemed of interest to obtain enough plates to determine the amplitude of variation and the relationship of the extremes of the light- and velocity-curves. The fifteen satisfactory plates now available are fairly well distributed over all phases of light variation and are sufficient for the present purpose. The data are listed in Table VIII with phases derived with formula (1).

The amplitude is small, and it is to be expected that the unavoidable errors will have a large percentage effect. The distribution of

[1] *Mt. Wilson Contr.*, No. 105, p. 12; *Astrophysical Journal*, **42**, 172, 1915.

the velocities when plotted with the phases in Table VIII is shown by the circles at the top of Figure 3. Barred circles represent the three velocities obtained in 1913.

These three early values, especially the first, would perhaps be more consistent with those of 1929–1930 were the period shortened by $0^d.00017$. The improvement, however, would be slight; moreover, for either period, some of the other velocities are more discordant than the early ones. Huffer's period can therefore scarcely be in error by as much as $0^d.0002$.

The radial velocities reach a maximum of $+6$ km/sec. and a minimum of -9 km/sec. The former falls close to phase 0^d, or maximum

TABLE VIII
RADIAL VELOCITIES OF DT CYGNI

Plate No.	Date	G.M.T.	Phase	Vel.
				km/sec.
γ 2726..........	1913 Oct. 10	16^h12^m	$2^d.332$	$- 2.2$
2829..........	Nov. 7	16 49	0.367	$- 7.3$
2936..........	Dec. 7	14 20	0.271	$- 8.6$
16647..........	1929 June 18	23 44	1.751	$+ 8.0$
16725..........	July 18	21 29	1.664	$+ 2.5$
16734..........	July 20	21 2	1.147	$+ 3.7$
16742..........	July 21	21 50	2.181	-10.0
16748..........	July 22	20 30	0.626	$+ 5.0$
16816..........	Aug. 18	15 33	2.427	$- 7.6$
16824..........	Aug. 19	19 28	1.090	$+ 5.1$
16835..........	Aug. 20	22 41	2.224	$- 5.5$
16901..........	Sept. 12	15 8	2.416	$- 5.9$
17023..........	Oct. 21	15 59	1.463	$+ 5.3$
17033..........	Oct. 22	17 13	0.015	$+ 0.2$
17501..........	1930 June 7	21 53	0.773	$- 0.6$

light. No very definite phase for maximum velocity is revealed by the plot, but a sine-curve symmetrical about the mean axis -0.5 km/sec. (velocity of the system) with its minimum at phase 0^d, maximum at phase $1^d.25$, and an amplitude of 15 km/sec. appears to satisfy the observations as well as might be expected. Such a curve has therefore been adopted and is drawn through the plotted velocities.

The results therefore indicate that Huffer's period needs very little, if any, revision, and that the velocity-curve is substantially a mirror image of the light-curve, the relation typical of Cepheid variables.

CARNEGIE INSTITUTION OF WASHINGTON
MOUNT WILSON OBSERVATORY
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TWO HIGH-SPEED CAMERA OBJECTIVES FOR ASTRONOMICAL SPECTROGRAPHS

W. B. RAYTON

ABSTRACT

Two new objectives of large relative aperture are described: (*a*) a magnified and somewhat modified copy of a 16.0-mm microscope objective having a focal length of 5 inches and relative aperture f/2.4; (*b*) a modified copy of a 4.0-mm microscope objective of numerical aperture 0.85 having an equivalent focus of 32 mm and relative aperture f/0.59.

It is, of course, a well-known fact that because of the small number of large telescopes available any advance in instruments or technique which will decrease the time required for a single observation is enthusiastically welcomed by the astronomer. The great light-gathering power of the 100-inch reflector at Mount Wilson makes it pre-eminently useful for the study of the spectra of faint nebulae, for example, but there is only one such telescope and the exposure-time required painfully limits the amount of work of this kind which can be done with it. For this reason the Observatory Council of the California Institute of Technology is making an extensive study of all instruments and auxiliary devices that will multiply the power of the proposed 200-inch reflector and other large telescopes. Accordingly, Dr. J. A. Anderson, executive officer of the Observatory Council, inquired in the fall of 1928 as to the possibilities of supplying short-focus camera objectives of very great relative aperture combined with excellent definition.

The fields of views required were small in comparison with those usually demanded of photographic objectives, making the problem more or less similar to that faced in designing objectives for microscopes. It therefore seemed worth while to try some experiments along this line.

The first lens tried was of 5-inch equivalent focal length and a speed of f/2.0. At this focal length the lens proved to have too much zonal spherical aberration: therefore the aperture was reduced later to f/2.4 and the lens refigured. In this form it is giving entirely satisfactory results with the 100-inch Mount Wilson reflector. The

59

type of construction employed is shown in Figure 1 with curves of spherical aberration. This lens is similar in construction to the ordinary 16-mm microscope objective.

Some months later Dr. Anderson raised the question whether, in a lens of shorter focal length, it would not be possible to attain much greater speed with the type of lens used in 4.0-mm microscope objectives. We accordingly made for trial at Mount Wilson a lens of equivalent focus 32 mm and of relative aperture f/0.59, similar to a 4-mm microscope objective of numerical aperture 0.85, with an extra element added to correct the lens for an infinite distance of object. This extra element could undoubtedly be dispensed with, but it did not then seem justifiable to spend the time required in computation without more assurance that the results of the experi-
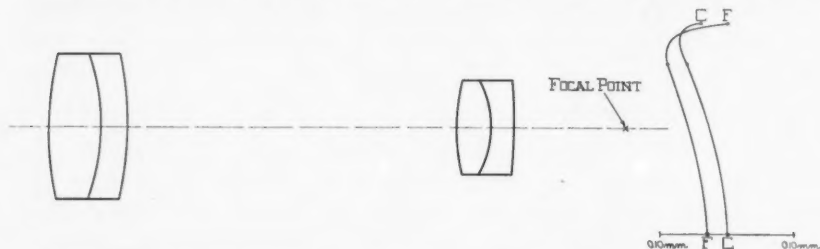

Fig. 1.—Camera objective $f = 5$ inches, $f/a = 2.4$, after design of 16-mm microscope objective.

ment were going to be of any value. The type of lens used in this case is shown in Figure 2. Curves of spherical aberration for three colors are included in the figure. It will be noticed at once that the distance from the lens to the plate is exceedingly small.

The correction of chromatic aberration is not particularly important in either lens since each color comes to a separate focus under any circumstances and best results follow from experimentally tilting the plate until the best average definition of the spectral lines is obtained. In both lenses spherical zones are greater in the more refrangible end of the spectrum, as in the usual case. The sine condition was given careful attention so as to avoid unsymmetrical flare on the spectral lines.

There is little doubt that the f/0.59 is the fastest lens which has ever been used as a spectrographic camera objective. Tests made

by Mr. Milton L. Humason with the 100-inch telescope show excellent definition and a gain of 50 per cent in speed over the short-focus camera objective previously used. Results obtained in photo-

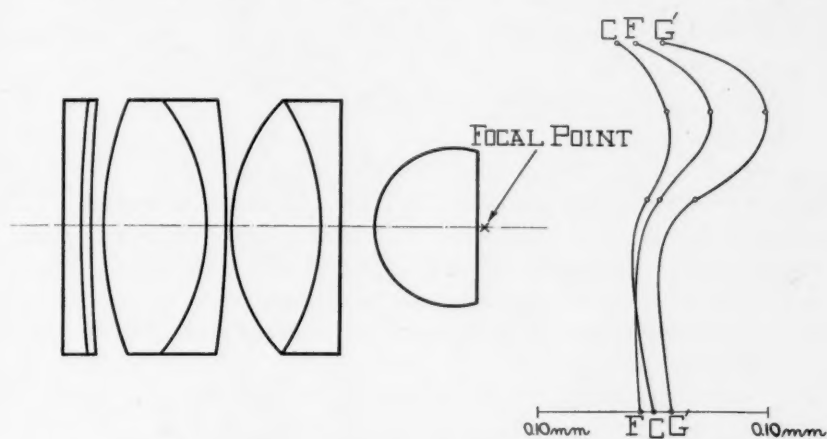FIG. 2.—Camera objective $f = 32$ mm, $f/a = 0.59$, after design of 4-mm microscope objective.

graphing the spectra of very remote spiral nebulae may be found in a recent paper by Mr. Humason.[1]

SCIENTIFIC BUREAU
BAUSCH & LOMB OPTICAL COMPANY
ROCHESTER, N.Y.

[1] "The Rayton Short-Focus Spectrographic Objective," *Mount Wilson Contributions*, No. 400; *Astrophysical Journal*, **71**, 351, 1930.

REVIEWS

The Structure of Line Spectra. By LINUS PAULING and SAMUEL GOUDSMIT. "International Series in Physics." Pp. x +263. $3.50. New York: McGraw-Hill Co., 1930.

This is a textbook on line spectra; it aims at a description of the great body of spectral data and of the semi-empirical rules which have made its systematic classification possible. The more general physical principles which are now believed to underly these rules are, quite justifiably, given only a brief reference. On the other hand, the book is not a mere compilation of data, its object being attained by the analysis of a few typical spectra in terms of the "vector" model, the model of penetrating orbits, of the spinning electron, etc.

After these models have been explained by a detailed consideration of alkali-like spectra, the extension to the general case of two-electron spectra (with general coupling), and to the Russell-Saunders coupling of many electrons, is readily accomplished. The intensity and polarization rules are adequately treated and the Pauli exclusion principle, with its explanation of the periodic system of elements, is introduced. The laws of X-ray spectra are accorded a treatment the relative brevity of which may invite the astrophysicist to examine this fundamental field of atomic physics, even though its data do not enter directly into his own problems.

To the spectroscopist, the chapter on the hyperfine structure (formerly called the "satellite structure") of spectral lines will be of great interest. This phenomenon has fascinated spectroscopists ever since its discovery by Michelson, because of the high resolution necessary for its observation but despite the considerable body of data which has accumulated, relatively little progress toward its systematic interpretation has been made. The work of Back and Goudsmit, in 1927–1928, on the hyperfine structure and Zeeman effect of the bismuth lines inaugurated a new phase of this study which promises to yield valuable clues to the structure of the nucleus.

The book closes with a chapter on magnetic phenomena other than the Zeeman effect. There are appendices, in which are to be found the numerical constants of spectroscopy and detailed accounts of the spectra of neon and iron. The authors have supplied many references to original papers in the form of footnotes.

CARL ECKART

RYERSON LABORATORY

Molecular Spectra and Molecular Structure. By the FARADAY SO-
CIETY. A general discussion. 1929. Pp. 346. 15s. 6d.

The papers published under the foregoing title were read at the sym-
posium of the Faraday Society in September, 1929. Thirty-eight con-
tributors, including practically all the well-known band spectroscopists,
participated in this discussion.

The book opens with a General Introduction, in which the problems
to be discussed are reviewed. The contributions are divided into three
parts: Part 1, "Band Spectra in the Visible and Ultra Violet Regions,"
dealing mostly with the spectra of diatomic molecules; Part 2, "The
Raman Effect," pertaining to the Raman effect in liquids and gases,
polarization of the scattered light, experimental procedure, etc.; and Part
3, "The Infra-Red Spectra of Solids, Liquids and Gases." A general dis-
cussion is given at the end of each division. The volume closes with a
summary covering the most important points in the various papers.

Band spectroscopy has been, and perhaps still is, considered to be
highly specialized and intricate. However, it is doubtful if it is more com-
plicated than line spectroscopy was considered to be five or six years ago.
Many empirical relations and assumptions, formerly puzzling to most
spectroscopists, have been explained successfully by the application of the
new quantum mechanics. Another source of confusion will disappear with
the adoption of a standard nomenclature, such as the one proposed in one
of the papers.

To the astrophysicist, dealing as he does with diatomic molecules,
Part 1, which comprises almost half of the volume, will be of the greatest
interest. It includes both theoretical and experimental papers. Of espe-
cial value are those on "Band Spectra and Atomic Nuclei," "Chemical
Binding," "The Electronic Structure of Some Diatomic Molecules,"
"Determination of Heats of Dissociation by Means of Band Spectra,"
"Recent Work on Isotopes in Band Spectra," as well as papers on the
spectra of the molecules of hydrogen, helium, and *CO*. Most of the con-
tributions are accompanied by a bibliography or numerous references.

One of the chief merits of this work is that most of the papers are com-
plete summaries, as well as real contributions to the particular subject of
which they treat. As a reference-book, this volume should prove of con-
siderable value.

ANDREW CHRISTY

RYERSON LABORATORY

L'ancienne et la nouvelle théorie des quanta. By Eugène Bloch. Paris: Librairie Scientifique Hermann et Cie, 1930. 8vo. Pp. i+417. Figs. 42. Unbound, Fr. 90.

This new book on the quantum theory is based upon two courses of lectures given by the author at the Sorbonne in 1926–1927 and in 1928–1929. It is intended as a textbook for students and presupposes little special knowledge on the part of the reader. In fact, the ideas of the quantum theory are developed from the very beginning, so that it covers practically the whole field of this branch of physics. The exposition, too, is rather elementary and nearly all mathematical deductions are given *in extenso*. In a few cases where this was not possible, references to standard textbooks are given.

There are twenty chapters, of which the first ten are devoted to the older quantum theory of Bohr, Sommerfeld, and others. Chapters xi and xii form a natural transition to the new quantum mechanics, and cover the fundamental results of analytical mechanics and the correspondence principle. Chapter xiii is devoted to the work of de Broglie. In chapters xiv –xvi we find an exposition of the wave mechanics of Schrödinger. Chapters xvii and xviii give the elements of matrix mechanics; chapter xix explains the principle of indetermination; and the concluding chapter discusses the statistics of Bose-Einstein and of Fermi. There are appended two mathematical notes and a short bibliographical Index.

The author has not attempted to cover all new developments. The theory of the Stark effect, band spectra, and the electronic theory of metals are some of the subjects omitted. Nevertheless the book can be highly recommended as an up-to-date introductory course to the quantum theory.

O. Struve